Logic Worksheet – Finale

Your name:	Mark (out of 5):
Logic class (A–F):	
Logic class tutor:	

This worksheet contains five questions, of roughly the sort you might expect to find in the formal component of the Part IA logic exam. The rubric for the exam is unchanged from previous years, and will be as follows: you have three hours to do three questions, and either one or two of these questions must come from the formal section of the paper. (The other question(s) will be more philosophical in nature.)

You should therefore treat this worksheet as providing you with mock exam questions, and you should try to do them in exam-like conditions. Do not try to do all of them in one sitting, but do:

- give yourself exactly one hour per question;
- work somewhere without distractions (e.g. a library);
- not look anything up while you are doing the questions.

Once you have completed them, self-mark all except those marked with a star (those will be marked by your logic class marker). The solutions will be available at http://www.nottub.com/forallx.shtml. Correct your own work *in red*, for the marker to review. In the box below, write something that you now firmly understand, as a result of doing these exercises:

Understand:

And in this box, write something that you want to know more about:

Want to know more about:

How you fill out these boxes will help to guide your logic class.

Each question should take an hour. A perfect answer would receive a notional 100 points. The number in square brackets after each component of a question designates the number of points that a full and correct answer to that component would merit. Questions marked with a star will be marked by your logic class marker.

1. Attempt all parts of this question.
   (a) Carefully define the following notions: [30]
      1. Truth-functional connectives
      2. Object language
      3. Metalanguage
      4. Tautologically equivalent sentences
      5. Provably equivalent sentences

   (c) Which of the following claims are true, and which are false? Explain your answers. [70]
      1. No valid argument has a premise which is necessarily true.
      2. No valid argument has a conclusion which is necessarily false.
      3. Every valid argument with a false conclusion has at least one false premise.
      4. Every valid argument with a true conclusion has at least one true premise.
      5. If we add a new premise to a valid argument, the resulting argument is valid.
      6. If 'A, hence either B or C' is valid, then so is 'Neither C nor B, hence not-A'.
      7. If 'A, hence if B then C' is valid, then so is 'both not-A and not-C, hence not-B'.
      8. Suppose we have a valid argument, with premises $\mathcal{A}_1, \ldots, \mathcal{A}_n$, and conclusion $\mathcal{C}$; then the argument with premises $\mathcal{A}_1, \ldots, \mathcal{A}_n$ and conclusion $\mathcal{B} \vee \mathcal{C}$ is also valid (whatever $\mathcal{B}$ might be).
      9. If $\vDash \neg(\mathcal{A} \to \mathcal{B})$, then $\mathcal{A}$ is not contingent.
      10. If $\mathcal{A} \vdash \mathcal{B}$, then $\mathcal{B}$ is not contingent.

      ★ Any consistent set of sentences can be rendered inconsistent, by the addition of a new sentence.

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2. Attempt all parts of this question.
   (a) Formalise each of the following arguments in FOL as best you can. You must provide a symbolization key for each argument: [25]
      1. Everyone loves someone. So at least two people love each other.
      2. Everyone is an existentialist iff everyone is a poet. So Camus is an existentialist iff he is a poet.
      3. The author of *L'Étranger* also wrote *Le mythe de Sisyphe*. So, at least one person wrote *Le mythe de Sisyphe*.

- ★ Everybody loves my baby, but my baby don't love nobody but me. So I am my baby.
- ★ Every dog is a mammal. So every bark produced by a dog is a bark produced by a mammal.

(b) For each of the arguments you formalised in part (a) of this question, state whether it is valid or invalid. If it is valid, provide a natural deduction to show that it is valid. If it is invalid, provide a counterexample to show that it is invalid. [75]

3. Attempt all parts of this question. You must use the proof system from the textbook.

   (a) Show each of the following: [50]

   1. $\vdash P \rightarrow (Q \rightarrow P)$
   2. $\vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$

   ★ $\vdash (\neg P \rightarrow \neg Q) \rightarrow (Q \rightarrow P)$

   (b) Show both of the following: [50]

   1. $\forall x Fx \leftrightarrow \exists x Gx \vdash \forall x Fx \vee \forall x \neg Gx$

   ★ $\exists x Rax \rightarrow \exists x Rxa \vdash \exists x (Rab \rightarrow Rxa)$

4. Attempt all parts of this question.

   (a) Explain what it means to say that a relation is: [10]

   1. reflexive
   2. symmetric
   3. transitive
   4. an equivalence relation

   (b) We say that a relation, R, is *negatively transitive* if and only if $\forall x \forall y \forall z((\neg Rxy \wedge \neg Ryz) \rightarrow \neg Rxz)$. Of the following relations, say which are reflexive, symmetric, transitive, and negatively transitive on the domain of people: [50]

   1. x is y's biological mother
   2. x is y's only sibling
   3. x is no taller than y
   4. x is a biological ancestor of y

   ★ x and y share no biological ancestor

   (c) Define the following terms: [10]

   1. subset
   2. proper subset
   3. union
   4. intersection
   5. Cartesian product

   (d) Attempt all parts of this question: [30]

   1. Show that if A is a subset of B and B is a subset of C, then A is a subset of C.
   2. Does the statement in part d.1 of this question remain true, if we replace all instances of the word 'subset' with 'proper subset'? Explain your answer.

★ Show that $A = B$ iff neither A nor B is a proper subset of $A \cup B$. Which axiom of set theory did you need to invoke to show this?

5. Attempt all parts of this question.

   (a) Explain each of the following notions from probability theory: [10]

      1. Conditional Probability
      2. Independent Events
      3. Exclusive Events
      4. Bayes's Theorem

   (b) The inhabitants of Dance Island are staging their annual ballroom dance contest. For this, the inhabitants of the island are placed into dancing pairs, at random. [20]

      1. You meet a dancing pair. The lead dancer of that pair was born in June. What is the probability that they were both born in June?
      2. You meet another dancing pair. One of them was born in June. What is the probability that they were both born in June?

      Flag any assumptions that you have had to make in answering the question. Also, comment on the relationship between your answers to (b)1 and (b)2.

   (c) In the town of South Park, 20% of the population are children. 1% of the children have lice; among the rest of the population, 0.1% of them have lice. LNL (Lice or No Lice^{TM}) is a means for testing whether or not someone has a lice. If a person has lice, LNL will definitely say the person has lice. If a person does not have lice, there is a 5% chance that LNL will say (falsely) that the person does have lice (this is not affected by whether the test is performed on a child or on someone else). Calculate the probability that a randomly chosen inhabitant of South Park: [70]

      1. has lice
      2. is a child, given that they have lice
      3. has lice, given that LNL says that they have lice

      ★ has lice, given both that they are child and LNL says that they have lice