Logic Worksheet 6

Your name:	Mark (out of 5):
Logic class (A–F):	
Logic class tutor:	

Reading

§§31–33, 35 of `forall`χ`:Cambridge`.

Self-marked exercises

Do the following practice exercises from `forall`χ`:Cambridge`.

- §31 Part A; Part questions B 2, 3; Part C Barbara, Ferio; Part D Barbari, Fesapo; Part E questions 6, 7
- §32 Part C questions 3–6 (NB: this requires 8 proofs)
- §33 Part A questions 2, 4, 6, 9; Part B

Exercises marked with a '$\star$' will be covered in the lecture on the day before the worksheet is due. When you have completed them, carefully check your answers against the answers available at http://www.nottub.com/forallx.shtml. Correct your own work *in red*, for the marker to review. In the box below, write something that you now firmly understand, as a result of doing these exercises:

Understand:

And in this box, write something that you want to know more about:

Want to know more about:

How you fill out these boxes will help to guide your logic class.

Further exercises

A. Many lectures ago, I promised you that you would be able to prove (using logic alone) that if you have one thing, and another thing, then you have two things. This question *almost* delivers on that promise.

Show the following:

$$\exists x(Fx \wedge \forall y(Fy \to x = y)), \exists x(\neg Fx \wedge \forall y(\neg Fy \to x = y)) \vdash \exists x \exists y(\neg x = y \wedge \forall z(x = z \vee y = z))$$

Warning: this is tough, and deliberately so. The final answer will take you around 30 steps. Maybe you won't get an answer to it. That's fine; but try it for at least an hour. You'll learn *a lot* by playing around with it.

Start with this hint:
Think about your answers to §33, Part B.

If after 20 further minutes, you still haven't got anywhere, read this next hint:
The key to this proof is keeping control over your quantifiers. And very often in a formal proof, your manipulation of the quantifiers will be guided by how you would argue in natural language. So:

- We can gloss the two 'premises' as follows: 'exactly one thing is F' and 'exactly one thing is not-F'.
- We can gloss the 'conclusion' as: 'there are exactly two things'.

So what you first want to do is consider some object – let's say a – of which you can say that it's the only F. Then you want to consider some other object – let's say b – of which you can say that it's the only non-F. Now if you think about some arbitrary object – let's say c – you should be able to show that it must be identical to a or identical to b. And the conclusion should follow from there.