LECTURE 3

VERIFICATIONISM AND THE PARADOX OF KNOWABILITY

3.1 Verificationism

Strong Verifiability: 'A meaningful statement is one which in practice could be conclusively shown to be true or false.' (Misak 1995: 65; she doesn't endorse this)

- Then intuitionists should worry about the *meaningfulness* of some LEM-instances.
- On the contrary, the BHK-interpretation *defined* the meaning of a sentence of the form '$\mathcal{P} \vee \neg\mathcal{P}$' so that they are meaningless iff '$\mathcal{P}$' is meaningless.

Modal Verification. A sentence's meaning is given by what would count as its verification. This seems better. But what's a verification?
In the mathematical case: a proof (though, in what system?)
In the empirical case: we have the difficulties from last time

3.2 The Knowability Principle

Knowability Principle. $\mathcal{P} \rightarrow \Diamond\mathrm{K}\mathcal{P}$
Intuitionists are likely to endorse this Principle.
First, if truth is warranted assertibility, then any truth should be *knowable*.
Second, the Knowability Principle ensures the world is not 'too far out of reach'. Now, maybe Classicists aren't worried about LEM, because they think that sentences can have truth values utterly independently of our ability to know about them. But:
— that leads to awful scepticism.
— that divorces truth from human practice, leaving us alienated from our world
But how does '**K**' behave? Two suggested rules of inference, *Factivity* and *Distribution*:

m	$\mathbf{K}\mathcal{P}$			m	$\mathbf{K}(\mathcal{P} \wedge \mathcal{Q})$	
	$\mathcal{P}$	KE			$\mathbf{K}\mathcal{P} \wedge \mathbf{K}\mathcal{Q}$	K-dist

3.3 The Paradox of Knowability

1	$(\mathcal{A} \wedge \neg\mathbf{K}\mathcal{A}) \to \Diamond\mathbf{K}(\mathcal{A} \wedge \neg\mathbf{K}\mathcal{A})$	Knowability Principle
2	$\quad \mathbf{K}(\mathcal{A} \wedge \neg\mathbf{K}\mathcal{A})$	
3	$\quad \mathcal{A} \wedge \neg\mathbf{K}\mathcal{A}$	K-factivity 2
4	$\quad \neg\mathbf{K}\mathcal{A}$	∧E 3
5	$\quad \mathbf{K}\mathcal{A} \wedge \mathbf{K}\neg\mathbf{K}\mathcal{A}$	K-distribution 2
6	$\quad \mathbf{K}\mathcal{A}$	∧E 5
7	$\quad \bot$	⊥I 6, 4
8	$\neg\mathbf{K}(\mathcal{A} \wedge \neg\mathbf{K}\mathcal{A})$	¬I 2–7
9	$\Box\neg\mathbf{K}(\mathcal{A} \wedge \neg\mathbf{K}\mathcal{A})$	□I 8
10	$\neg\Diamond\mathbf{K}(\mathcal{A} \wedge \neg\mathbf{K}\mathcal{A})$	CM 9
11	$\quad \mathcal{A}$	
12	$\quad\quad \neg\mathbf{K}\mathcal{A}$	
13	$\quad\quad \mathcal{A} \wedge \neg\mathbf{K}\mathcal{A}$	∧I 11, 12
14	$\quad\quad \Diamond\mathbf{K}(\mathcal{A} \wedge \neg\mathbf{K}\mathcal{A})$	→E 1, 13
15	$\quad\quad \bot$	⊥I 14, 10
16	$\quad \neg\neg\mathbf{K}\mathcal{A}$	¬I 12–15
17	$\quad \mathbf{K}\mathcal{A}$	DNE 16
18	$\mathcal{A} \to \mathbf{K}\mathcal{A}$	¬I 11–17

So the Knowability Principle seems to entail:

Knowledge Principle. $\mathcal{A} \rightarrow \mathbf{K}\mathcal{A}$
This is *awful*. Surely there are plenty of things that no one will ever know. But it seems to be entailed by the (reasonable) Knowability Principle.

3.4 Response 1: restrict the Knowability Principle

One response is to *restrict* the instances of the Knowability Principle.
Φ-Knowability Principle. $\mathcal{P} \rightarrow \Diamond\mathbf{K}\mathcal{P}$, where statement $\mathcal{P}$ has property Φ.

Dummett (2001) once endorsed a particular version:
Basic-Knowability Principle. $\mathcal{P} \rightarrow \Diamond\mathbf{K}\mathcal{P}$, where $\mathcal{P}$ is basic
Truth is then defined in the standard recursive way, for complex statements:

'$\mathcal{P} \wedge \mathcal{Q}$' is true **iff** '$\mathcal{P}$' is true and '$\mathcal{Q}$' is true
'$\mathcal{P} \vee \mathcal{Q}$' is true **iff** '$\mathcal{P}$' is true or '$\mathcal{Q}$' is true
'$\mathcal{P} \rightarrow \mathcal{Q}$' is true **iff** if '$\mathcal{P}$' is true then '$\mathcal{Q}$' is true
'$\neg\mathcal{P}$' is true **iff** '$\mathcal{P}$' is not true

But why should the connectives on the right-side of these clauses be *intuitionistic*? 'By having confined the knowability principle to atomic statements, it would appear that Dummett has foregone the most important principled way... to argue against the illicit application of strictly classical rules of inference. No longer is he requiring of *every* proposition of arithmetic that, if it is true, then it is knowable.' (Tennant 2002: 141)

Tennant (1997, *The Taming of the True*) endorses:
Cartesian-Knowability Principle. $\mathcal{P} \rightarrow \Diamond\mathbf{K}\mathcal{P}$, where $\mathcal{P}$ is any statement where it is not logically impossible that $\mathbf{K}\mathcal{P}$.
Fitchian Unknowables tells us that $(\mathcal{P} \wedge \neg\mathbf{K}\mathcal{P})$ is not Cartesian (for any $\mathcal{P}$).
So this blocks the derivation of the Knowledge Principle.
But we might wonder whether the Cartesian-Knowability Principle isn't just a bit *ad hoc*.

3.5 Response 2: emphasise intuitionism

Dummett ('Reply to Künne' 2007, 'Fitch's Paradox of Knowability' 2009) emphasises that the proof of the Knowledge Principle is *intuitionistically invalid* (see also Williamson 1982). Look at line 17! Intuitionists can infer only:
Intuitionistic Knowability Principle. $\mathcal{P} \rightarrow \neg\neg\mathbf{K}\mathcal{P}$
What does '$\neg\neg\mathbf{K}\mathcal{P}$' mean for Dummett? Via BHK:

- Any warrant for $\neg\mathbf{K}\mathcal{P}$ can be converted to a warrant for absurdity.
- Any warrant for [any warrant for $\mathbf{K}\mathcal{P}$ can be converted to a warrant for absurdity] can be converted to a warrant for absurdity.

Dummett suggests that this just means that it is possible (in some sense) that $\mathbf{K}\mathcal{P}$.
So this is just an intuitionistic gloss on the original idea that all truths are knowable.
So perhaps the Knowability Principle and intuitionism are mutually reinforcing?

Problem 1. The following is an (intuitionistic) contradiction: $\mathcal{P} \wedge \neg\mathbf{K}\mathcal{P}$.
Reply: if truth amounts to warranted assertibility, that's absolutely correct!
But: is this any more than Moore's Paradox?

Problem 2. The following is an (intuitionistic) contradiction: $\neg\mathbf{K}\mathcal{P} \wedge \neg\mathbf{K}\neg\mathcal{P}$.
Reply: first, think about how '**K**' is being understood. (Not as 'it is now known that ...'.)
Then think about how an intuitionist understands '$\neg$'.
But: think about an empirical case.