Response of subglacial sediments to basal freeze-on
1. Theory and comparison to observations from beneath the West Antarctic Ice Sheet

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[1] We have constructed a high-resolution numerical model of heat, water, and solute flows in sub-ice stream till subjected to basal freeze-on. The model builds on quantitative treatments of frost heave in permafrost soils. The full version of the Clapeyron equation is used. Hence, ice-water phase transition depends on water pressure, osmotic pressure, and surface tension. The two latter effects can lead to supercooling of the ice base. This supercooling, in turn, induces hydraulic gradients that drive upward flow of pore water, which feeds the growth of segregation ice onto the freezing interface. This interface may progress into the till and form ice lenses if supercooling is sufficiently strong. Hence, the ice segregation process develops a stratified basal ice layer. In our model, a high basal temperature gradient (\(\frac{1}{C_24} 0.054\) \(\frac{1}{C_176} C m\)) triggers ice stream stoppage, and the loss of basal shear heating leads to relatively high basal freeze-on rates (\(\frac{1}{C_24} 3–5\) mm a\(^{-1}\)). In response, the subglacial till experiences comparatively rapid consolidation. Till porosity can decrease from 40% to <25%, and till strength can increase from \(\frac{1}{C_24} 3\) kPa to >120 kPa, approximately within one century. Basal supercooling arising from redistribution of solutes and ice-water interfacial effects amounts to ca. \(-0.35^\circ C\) below the pressure-melting point. Fine-grained till is in our model associated with widely spaced, thick ice lenses. Coarse-grained till yields thinner ice lenses that are more closely spaced. Our model results compare favorably, although not in all details, with available observational evidence from borehole studies of West Antarctic ice streams.

INDEX TERMS: 1823 Hydrology: Frozen ground; 1827 Hydrology: Glaciology (1863); 3210 Mathematical Geophysics: Modeling; 3220 Mathematical Geophysics: Nonlinear dynamics; KEYWORDS: Antarctica, ice stream, stoppage, freeze-on, basal ice, till


1. Introduction

[2] Weak tills beneath fast flowing glaciers and ice streams can act as a lubricating boundary layer separating the ice base and the bedrock [Alley et al., 1987; Engelhardt et al., 1990; Engelhardt and Kamb, 1998]. The subglacial presence of soft till may reduce the basal drag to a degree where the driving stress is supported mainly by the lateral shear margins [Echelmeyer et al., 1994; Jackson and Kamb, 1997; Raymond, 1996; Whillans and van der Veen, 1997]. Soft bed conditions are found beneath the ice streams of the West Antarctic Ice Sheet. The most studied ice streams in Antarctica are located in the Ross Sea sector, which is outlined in Figure 1. The existence of soft and porous till was initially inferred from geophysical surveys [Blankenship et al., 1986, 1987]. Subsequent drilling confirmed the presence of weak sub-ice stream till [Engelhardt et al., 1990; Engelhardt and Kamb, 1997, 1998; Kamb, 1991]. The physical properties of the till have been established through in-situ torvane measurements and laboratory analysis of core samples [Kamb, 2001a; Tulaczyk et al., 1998, 2000a]. Poorly drained subglacial conditions promote buildup of high sub-ice stream pore water pressures and low effective pressures (\(<1–2\) kPa) [Tulaczyk et al., 2001]. Hence, the till porosity is high (\(\approx 40\%\)) while the basal shear strength is low (\(<2–3\) kPa) [Kamb, 2001a]. The lubricating effect arises when the basal shear strength of the till drops below the driving stress, which is less than 15 kPa for the Whillans Ice Stream [Engelhardt and Kamb, 1998; Kamb, 1991]. Physical properties of subglacial tills are a vital control on the dynamics of soft-bedded ice streams, e.g., in the Ross Sea sector of the West Antarctic ice sheet [Joughin and Tulaczyk, 2002; Joughin et al., 2002; Kamb, 1991].
Development of quantitative theory and models of subglacial freezing is needed because the thermal history of the basal zone may be linked to important aspects of ice stream dynamics. Unlike basal melting, freeze-on leaves behind a physical record of its action in the form of basal ice layers. Such layers have been found in many of the deep boreholes drilled in modern polar ice sheets [Goodwin, 1993; Gow et al., 1979; Herron and Langway, 1979; Hooker et al., 1999; Koerner and Fisher, 1979; Siegert et al., 2000]. Recent borehole investigation with digital video camera at the base of Ice Stream C has shown up to 25-m-thick layers of debris-bearing and clear basal ice, which has been interpreted as a product of basal freeze-on [Carsey et al., 2003; Kamb, 2001b]. The properties of these accretion zones can help us interpret the thermal history of the basal regimen and the history of sub-ice sheet hydrology [Boulton and Spring, 1986; Hubbard, 1991; Hubbard and Sharp, 1995; Lawson and Kulla, 1977; Lawson et al., 1998].

Here we present a theoretical treatment of coupled heat, water, and solute transport in till subjected to basal freeze-on. Our treatment of these processes is based on an extensive body of empirical and theoretical work on freezing of sediments in permafrost environments [Fowler and Krantz, 1994; Konrad and Duquennoi, 1993; O’Neill and Miller, 1985]. Recent research on premelting of ice in porous media may further improve the existing understanding of sediment freezing [Rempel et al., 2001a; Rempel and Worster, 1999; Wettlaufer and Worster, 1995]. To make predictions regarding the evolution of subglacial till and basal ice during freezing we have formulated a numerical, vertical-column model of the freezing process. The model has a high spatial resolution (node spacing ≤0.01 m) and it is constrained by initial conditions and boundary conditions aimed at emulating the non-steady basal environment of an ice stream undergoing a transition from fast flow to stagnation. Whereas this paper focuses on detailed till evolution under conditions of basal freezing, the companion paper by Bougamont et al. [2003] discusses a model of ice stream stoppage driven by basal freezing. A complete coupling of the two methods was unfeasible due to computational limitations.

2. Assumed Physics of Basal Freeze-On

Direct investigations of basal freeze-on are rare because of the logistical challenges associated with studying such processes in their modern subglacial environment [Lawson et al., 1998]. Although basic theoretical treatments of this phenomenon have been introduced into glaciology several decades ago [Weertman, 1961], there is a paucity of theoretical and empirical investigations of heat, water, and solute flow during basal freeze-on. To develop our model we have assumed that basal freeze-on resembles the phenomenon of frost heave that has been studied extensively by permafrost engineers during the last several decades [Fowler and Krantz, 1994; Konrad and Duquennoi, 1993; Miyata, 1998; O’Neill and Miller, 1985]. This assumption is supported by the general macroscopic similarity of ice formed by basal freeze-on and by frost heaving, as seen in Figure 2. This figure contains images of basal ice from Ice Stream C (Figures 2a and 2b) obtained from a borehole camera system [Carsey et al., 2003] as well as the outcome

Figure 1. Satellite image of the Ross Sea sector of the West Antarctic Ice Sheet showing the location of Whillans Ice Stream (WIS), Ice Stream C (ISC) and Ice Stream D (ISD). Labels UpB, UpC and UpD designate areas in which basal conditions have been studied through borehole experiments and sampling [Kamb, 2001a]. The background image has been generated using AVHRR data distributed by the USGS office in Flagstaff. Inset in the lower left corner shows a shaded relief image of the grounded portions of Antarctic ice sheet. White box gives approximate extent of the area shown in the main figure.

2001a]. The companion paper by Bougamont et al. [2003] features in-depth numerical investigation of basal freeze-on in relation to ice stream dynamics.

Tulaczyk et al. [2000b] conjectured that basal freeze-on may be the primary process through which activity of ice streams is terminated. The shutdown of Ice Stream C, approximately 150 years BP, may have been caused by freeze-on-driven consolidation of subglacial till [Bougamont et al., 2003]. In general, basal freezing may result from climatic cooling, ice thinning, decrease in basal shear heating or a combination of these factors [Alley et al., 1997]. Previous investigations of subglacial hydrology and till consolidation focused mainly on the condition of basal melting [Boulton et al., 1995; Boulton and Dobbie, 1993; Piotrowski and Kraus, 1997]. Borehole measurements in two active ice streams (Whillans Ice Stream and Ice Stream D) yielded basal temperatures at the pressure melting point. However, basal temperature of the stagnant Ice Stream C is −0.35°C below the pressure melting point although the underlying fine-grained till remains unfrozen [Bentley et al., 1998; Kamb, 2001a]. The bed of interstream ridges is typically frozen [Bentley et al., 1998; Gades et al., 2000; Kamb, 2001a]. Previous modeling studies have suggested that the lateral boundaries of ice streams correspond to a transition between basal melting and basal freezing [Jacobson and Raymond, 1998; Raymond, 1996], although the availability of sediments may also modulate margin location [Anandakrishnan et al., 1998; Bell et al., 1998].
of an experimental frost heave study (Figure 2c) [Watanabe et al., 2001]. Although there are some stratigraphic differences between sub-ice sheet freeze-on and ice segregation generated in an idealized porous medium, a fundamental macroscopic similarity clearly exist. Secondary differences in interlayering may be caused by very large differences (~orders of magnitude) in freeze rate and temperature gradients between the sub-ice stream conditions and the laboratory experiments of Watanabe et al. [2001].

Frost heaving occurs when soil freezing induces water flow and volumetric expansion beyond that caused by the mere expansion of water on freezing [O’Neill and Miller, 1985]. Whether given sediments are susceptible to frost heaving and segregation ice growth depends on grain-size distribution [Everett, 1961; Hohmann, 1997; O’Neill, 1983; Tester and Gaskin, 1996]. Whereas fundamental physics of basal freeze-on and frost heave may be similar, the physical setting of the subglacial environment is different from a typical permafrost setting. Frost heave is a seasonal surface process in which the overburden pressure is typically small (~10–100 kPa) and the vertical temperature gradients are typically large (~1–10°C m⁻¹). The overburden ice pressure acting during subglacial freezing is very large (~1–10 MPa), and vertical temperature gradients in basal ice are relatively small (~0.01–0.1°C m⁻¹). Hence, subglacial freezing rates will be slow compared to the freezing rates observed in near-surface permafrost processes. However, we expect that the long timescale over which subglacial sediments may be exposed to freezing (~100s–1000s years) provides basis for significant freeze-induced pressure changes in the basal environment.

We do not include a basal water system in our numerical simulations. Instead, we use the end-member assumption of entirely local hydrological balance, with no loss or gain of water from long-distance transport in a basal water system. We are not claiming conclusively that long-distance basal water transport is indeed negligible beneath the West Antarctic ice streams. The problem of existence and physical nature of such long-distance transport is still open to interpretation. Much previous research has emphasized the importance of a distributed, throughgoing basal water system beneath ice streams, e.g. in lubricating their beds and supplying latent heat to areas of basal freezing [e.g., Alley et al., 1994]. Theoretical analysis [Weertman and Birchfield, 1982; Walder and Fowler, 1994] and scaled physical models [Catania and Paola, 2001] indicate that any such water system, if it exists, should remain widespread. Recent numerical modeling has addressed the question whether a regional basal water system must exist beneath the West Antarctic ice streams by estimating the regional net balance of basal melting and freezing. Constraints on the magnitude of basal shear heating and geothermal flux are, however, so insufficient that it is possible to calculate either large net melting or net freezing rates for the same parts of the ice stream system [Parizek et al., 2002; Joughin et al., 2003; Vogel et al., 2003].

Borehole studies of sub-ice stream hydrology yielded many important observations that are, nonetheless, often difficult to interpret or even contradictory [Kamb, 2001a]. In his overview of borehole observations from West Antarctica, Kamb [2001a, 2001b, sections 9.2 and 9.3] provided a detailed discussion of the undrained-bed model and concluded that it offers a useful framework for understanding ice stream dynamics in general, and for explaining the stoppage of Ice Stream C in particular. A key limitation of borehole studies is that they necessarily sample small spatial areas. In our opinion, the most complete view of sub-ice streambeds comes from a large quantity of geophysical and sedimentological data acquired in the Ross Sea. The data have widespread regional coverage [Shipp et al., 1999, Figure 2] with high horizontal resolution (down to ~1 m; ibid. p. 1512) and include areas over which the West Antarctic ice streams extended during the Last Glacial Maximum (LGM). Anderson [1999, p.72] has summarized relevant observations in the following passage: “Notably absent in the ... records from the Ross Sea floor are tunnel valleys, subglacial braided channels, outwash fans/deltas, and eskers, which would imply channeled subglacial meltwater. In addition, hundreds of piston cores ... have only on rare occasions recovered graded sands and gravels that might be associated with subglacial meltwater systems.
The few exceptions ... are cores acquired near the termini of valley and outlet glaciers. The virtual absence of meltwater features and deposits ... is perhaps the most important difference in geomorphic character between the Antarctic continental shelf and Northern Hemisphere glacial terrains."

[10] Notwithstanding the controversy regarding the existence and nature of sub-ice stream water drainage, we adhere to the undrained assumption. We do so to generate an end-member view of ice streamflow. We consider it possible that the undrained model is the best description of the ice streams. Further, we note from the work of Parizek et al. [2002] that an undrained model likely was even more applicable in the past. Regardless, the reader should bear in mind that we are working on an end-member of possible ice stream behavior.

2.1. Ice-Water Interface Curvature and Surface Tension Effects

[11] Phase changes in an ice-water system are typically thought of as being controlled mainly by temperature and pressure. In the glaciological literature, the concept of the pressure-melting point is commonly utilized as a synonym for the freezing point. However, there are other less commonly considered factors that influence the temperature at which the ice-water phase transition occurs. One such factor is the presence of solutes in the liquid water. An increase in solute concentration has the same effect as an increase in fluid pressure as it depresses the freezing point. The influence of solute concentration can be expressed formally through a pressure term, which is referred to as the osmotic pressure [Padilla and Villeneuve, 1992]. Another factor, which is often overlooked, is ice-water interfacial effects, especially surface tension arising from interface curvature. This factor becomes important when ice crystal growth is restricted to fine inter-crystalline veins [Harrison, 1972; Raymond and Harrison, 1975] or micron-sized pore spaces of fine-grained subglacial sediments [Tulaczyk, 1999].

The surface tension effect is paramount in setting up a hydraulic gradient that drives water flow in a freezing porous media [Everett, 1961]. In general, the finer grained a sediment is, the higher is the curvature of ice-water interfaces, and the greater is the depression of the ice-water phase change temperature, i.e. the freezing point [Hohmann, 1997].

Unfrozen water has been observed in clays at temperatures lower than −10°C [O’Neill, 1983].

[12] When liquid water and ice co-exist in a curved interface configuration, there is a pressure jump between the two phases due to the interfacial effects [Fowler and Krantz, 1994; O’Neill and Miller, 1985]. The size of the pressure jump depends on the curvature of the ice-water interface as proposed by Gold [1957]. The most simplified assumption for the interfacial pressure jump between $p_i$ and $p_w$ is [Everett, 1961; Hopke, 1980; Tulaczyk, 1999]:

$$p_i - p_w = \sigma_{iw} \frac{dA}{dV}$$  \hspace{1cm} (1a)

where $p_i$ is the ice pressure, $p_w$ is the pore water pressure, $\sigma_{iw}$ is the ice-water surface energy, $dA/dV$ is the curvature of the ice-water interface. At the ice-till interface, $dA/dV$ is a function of the effective pressure. At zero effective stress, $dA/dV = 0$ and the ice base is planar. If the effective pressure reaches a critical value, the ice-water interface complies with the particle surfaces and its structure should be numerically equal to the specific surface area of the sediments, $SSA$. The ice-water curvature is in this case $dA/dV = SSA$ [Tulaczyk, 1999].

The specific surface area is small if the sediment is coarse-grained and ice may freely intrude the pore spaces. If the sediment is fine-grained, the specific surface area is high and the ice-water interface has a high curvature, which impedes ice formation. This effect can be ascribed, at least formally, to an interfacial effective pressure [Tulaczyk, 1999]:

$$p_{iw} = p_i - p_w = \sigma_{iw} SSA \frac{1}{r_p}$$  \hspace{1cm} (1b)

where subscript ‘iw’ refers to the ice-water interface, $SSA$ is specific surface area, and $r_p$ is the characteristic particle radius.

[13] Freezing of liquid water takes place when the pressure components and the temperature satisfies the generalized form of the Clapeyron equation [Fowler and Krantz, 1994; Miyata, 1998; O’Neill and Miller, 1985]. When solutes are present in the liquid water, the generalized Clapeyron equation becomes [Padilla and Villeneuve, 1992]:

$$\frac{p_{iw} - p_i}{p_w} = \frac{L}{273.15} T + \frac{p_0}{p_w}$$  \hspace{1cm} (2)

where $p_{iw}$ is the water pressure, $p_i$ is the ice pressure, $p_o$ is the osmotic pressure, $p_w$ is the density of water, $r_i$ is the density of ice, $L$ is the coefficient of latent heat of fusion and $T$ is the temperature in °C. This form of the Clapeyron equation represents a general thermodynamic relation whose validity is not limited to our specific purpose. Its validity has been verified experimentally [Biermans et al., 1978; Konrad and Duquennoi, 1993; Krantz and Adams, 1996; Miyata and Akagawa, 1998] and it provides the fundamental basis for frost heave models [Fowler and Krantz, 1994; Miyata, 1998; O’Neill and Miller, 1985]. The Clapeyron equation provides a mean for coupling pressure terms and temperature in a freezing porous medium. An examination of this equation demonstrates that a significant pressure jump can develop across the ice-water interface when supercooled liquid pore water is present beneath the freezing interface.

[14] The effect of interface curvature, equation (1a) or (1b), and the Clapeyron equation (2), can be linked together. Solving the former for ice pressure gives $p_i = 1/r_p + p_{iw}$, and insertion into the latter yields:

$$T = -\frac{273.15}{L} \left( \frac{1}{\rho_i} - \frac{1}{\rho_w} \right) p_{iw} - \frac{273.15 \sigma_{iw}}{L \rho_p} - \frac{273.15}{\rho_w} p_o$$  \hspace{1cm} (3)

This equation is the same expression used by Raymond and Harrison [1975] to treat freezing of water in micron-sized veins between ice crystals. It is also the fundamental equation for the temperature of ice-water phase transition given in Hooke [1998, p. 5]. The first term of equation (3) specifies the effect of water pressure on phase transition.
coupled flows of water, heat and solutes. Important feed-
backs arise because the flow of water initialized by cooling 
may carry sufficient heat and solutes to significantly influ-
ence the freezing conditions at the ice base.

Vertical water flow through unfractured, porous 
media occurs in response to a gradient in excess water 
pressure. The excess water pressure, $p_w$, is defined as the 
water pressure component in excess of an initial hydrostatic 
pressure, $p_h$. The total water pressure is thus $p_w = p_h + u$ 
[Domenico and Schwartz, 1990, equation (4.50)]. From 
Darcy’s law, vertical water flow velocity is assumed to be 
proportional to the excess water pressure gradient, $\partial u/\partial z$ 
[Domenico and Schwartz, 1990, equation (4.53)]:

$$
\nu_w = -\frac{K_h}{\rho_w g} \frac{\partial u}{\partial z}
$$

where $\nu_w$ is the water flow velocity, $K_h$ is the coefficient of 
hydraulic conductivity, $\rho_w$ is the density of water, and $g$ is 
the acceleration of gravity. Vertical gradients in excess pore 
pressure that build up in a freezing till can be obtained by 
solving a one-dimensional diffusion equation [Mitchell, 
1993, equation (13.19)]:

$$
\frac{\partial u}{\partial t} = c_r \frac{\partial^2 u}{\partial z^2}
$$

where $t$ is time, $c_r$ is the hydraulic diffusion coefficient, and $z$ is the depth coordinate (taken here to be zero at the ice 
base). Once water flow rates are determined, vertical 
transport of heat can be derived from a diffusion-advection 
equation [Domenico and Schwartz, 1990, equation (9.21)]:

$$
\frac{\partial T}{\partial t} = c_T \frac{\partial^2 T}{\partial z^2} - \nu_w \frac{\partial T}{\partial z}
$$

where $T$ is temperature, $c_T$ is the thermal diffusion 
coefficient, and $\nu_w$ is the velocity of water flow. Vertical 
transport of solutes in subglacial sediments underlying a 
frozen ice base can be determined from an analogous 
diffusion-advection equation [Domenico and Schwartz, 
1990, equation (13.9)]:

$$
\frac{\partial C}{\partial t} = c_c \frac{\partial^2 C}{\partial z^2} - \nu_w \frac{\partial C}{\partial z}
$$

where $C$ is the concentration of solutes and $c_c$ is the 
chemical diffusion coefficient. We neglect the influence of 
convection on heat and solute redistribution because the 
Rayleigh number for the considered problem is several 
orders of magnitude smaller than the usual convection 
threshold [Domenico and Schwartz, 1990, equation (9.25)].

2.3. Formation of Ice Lenses

Liquid water flows toward the freezing interface 
where it accretes into a layer of segregation ice. This layer 
forms as long as the surface-tension penalty for forming ice 
crystals in small pore spaces is large enough to suppress 
growth of ice within the till [Everett, 1961; Konrad and 
Duquennoi, 1993; Miyata, 1998; O’Neill and Miller, 1985]. 
When the distributions of water pressure, temperature and 
solute concentration are known, one can rearrange the
Clapeyron equation (2), and solve it for ice pressure within the till pore spaces:

\[ p_i = \frac{p_n}{p_w} (p_w - p_n) - \frac{T}{273.15} p_w L \]

This pressure should exist inside any small ice crystal that may form in the confined pore space of fine-grained sediments. When the ice pressure exceeds the sum of gravitational overburden pressure and the ice-water interfacial pressure, nothing is left to keep the crystal from growing beyond the confines of the pore space in which it initially formed [O’Neill and Miller, 1985]. For ice lens initiation, we thus use the criterion [Hopke, 1980]:

\[ p_i \geq p_n + p_{iw} \]

where \( p_n \) is the vertical overburden pressure and \( p_{iw} \) is the ice-water interfacial pressure. An ice lens grows through accretion of segregation ice until a new lens is initiated. The complex dependence of ice pressure on temperature, water pressure, and osmotic pressure determines where within the till there will be a new, thermodynamically more favorable location for ice crystal growth (equation (9)). Through this process of progressive ice-lens formation, the freezing front moves into increasingly deep levels in the till. This mechanism produces the banded ice-sediment structure of freezing soils [Fowler and Krantz, 1994; Konrad, 1994], and it may play an equally important role in forming layered, debris-bearing basal ice [Gow et al., 1979; Herron and Langway, 1979; Koerner and Fisher, 1979; Lawson et al., 1998]. This concept of subglacial ice lens development is shown in Figure 4.

Ice lens initiation is a sensitive and particularly complicated part of frost heave simulations that typically involves an empirical treatment as in O’Neill and Miller [1985]. A micro-scale approach to frost heave analysis related to premelting of ice [Wettlaufer and Worster, 1995; Wilen and Dash, 1995] and thermomolecular force [Rempel et al., 2001b] may lead to a more complete thermodynamical treatment. However, macro-scale models can be tested and verified via experimental studies [Fowler and Krantz, 1994; Krantz and Adams, 1996; Miyata, 1998; Miyata and Akagawa, 1998] and they can also be compared to field observations [Hohmann, 1997; O’Neill, 1983]. There is thus a major advantage in the phenomenological use of the interfacial effects and phase equilibria [Michalowski, 1993]. The method proposed by O’Neill and Miller [1985], on which we base our work, is the most commonly used theoretical treatment of contemporary frost heave models.

Force balance associated with ice pressure acting in growing ice lenses and effective pressure and water pressure acting in the surrounding unfrozen till is shown conceptually in Figure 5. Incompletely frozen till can become trapped between two neighboring ice lenses and this may effectively isolate the till from further inflow/outflow of water and solutes. These inactive till layers separated by segregation ice should, after sufficient cooling, freeze completely and become fully incorporated into the basal ice.

2.4. Regelation

Previous models of ice intrusion into subglacial sediments have concentrated on the process of regelation, e.g., Iverson [1993] and Iverson and Semmens [1995].

Figure 4. Schematic diagram showing the principal stages of basal freeze-on: (a) pore water flows toward the ice base in response to freezing, (b) pore water accretes onto the ice base as a layer of segregation ice, (c) the freezing front moves into the till and an ice lens develops, and (d) a second ice lens develops deeper in the till. A thin veneer of ice regelates into the till beneath ice lenses.
beneath glaciers is thermodynamically similar. However, our model is not simply a frost heave model. Creating a glaciological context required us to incorporate several glaciological parameters and variables, e.g. ice stream velocity and time-dependent frictional heat.

3. Modeling Approach

[22] The numerical model that we present here was generated to explore evolution of till properties during basal freeze-on. We utilize concepts related to contemporary frost heave models because we believe that basal freeze-on

Surface tension effects are negligible in coarse-grained sediments and the pressure that opposes intrusion of ice into pore spaces is the pore water pressure alone. For regelation into fine-grained sediments ice pressure must exceed the pore water pressure as well as the interfacial tension [Tulaczyk, 1999]. Modification of equation (1b) yields the critical subglacial water pressure below which ice may intrude into the pore spaces of the underlying till:

\[ p_{w, crit} = p_n - \sigma_{in} \]  

(10)

Once regelation starts, the velocity of regelation is finite and we use an experimentally based formulation given by Iverson and Semmens [1995]:

\[ V_r = K_r \frac{\Delta p}{z_r} \]  

(11)

where \( K_r \) is the conductivity of the sediment to ice, and \( \Delta p = p_i - p_w \) is the driving stress for regelation, and \( z_r \) is the vertical penetration depth of ice.

3. Model Configuration and Numerical Approach

[23] The model domain consists of a vertical till column of finite thickness (typically several meters). The column is overlain by ice and underlain by some arbitrary substratum (e.g. bedrock or sediment) that is not incorporated explicitly but treated as a boundary condition. The three fundamental variables determining the evolution of a till layer during freezing are excess pore pressure (\( u \)), temperature (\( T \)) and solute concentration (\( C \)). By using a vertical column model we make the implicit assumption that horizontal diffusion and advection of heat, water, and solutes are negligible. The simulated system is not only coupled but also rendered strongly non-linear by introduction of threshold criteria, such as the criterion for new ice-lens formation (equation (9)). The pressure components of the system change with time and they are coupled to the temperature evolution through the Clapeyron equation (2). We use high spatial resolution (\( \leq 0.01 \text{ m} \)) to represent the freezing front that moves down through the till column in discrete jumps.

[24] In order to put our modeling effort into a realistic glaciological context, the freezing till column is coupled to a highly simplified ice stream model based on the treatment of Tulaczyk et al. [2000b]. The numerical till column provides the basal shear stress (i.e., time-dependent till strength) that is needed to calculate ice stream velocity. In return, the analytical ice stream module passes to the till column the ice sliding velocity that is needed to determine the magnitude of shear heating. To investigate the potential influence of different hydrogeologic settings [Boulton and Dobbie, 1993], we consider two end-member cases for the lower boundary condition. The first case is a closed water system that simulates an impermeable substratum below the till. There is no influx of water into the till in this case and water can only redistribute itself within the till in response to freezing. The second case is an open water system that simulates a permeable, water-bearing substratum below the till. In this case water can enter the till from the underlying groundwater system.

[25] Configuration of the model domain is illustrated schematically in Figure 6. Complexity of the simulated system requires short time steps in order to avoid numeric instability. In a typical run, we used a time step of 20 minutes min while running the model for several hundreds of years. We applied the finite difference, forward Euler method to approximate the partial derivatives present in our system of equations. The code was developed on a double-processor Sun Ultra 80 workstation, but the final results were obtained on SUN Fire 6800 servers, which are part of High Performance Computing hardware at the Technical University of Denmark.

3.2. Coupling of the Numerical Till Model to an Analytical Ice Stream Model

[26] Our coupled ice-till model has been calibrated to simulate freezing conditions beneath modern West Antarctic
ice streams. This approach is justified by the fact that the subglacial zones of these till-bedded ice streams have been extensively studied through borehole observations and geophysical surveys [Bentley, 1998; Bentley et al., 1998; Blankenship et al., 1986, 1987; Engelhardt et al., 1990; Engelhardt and Kamb, 1997, 1998; Tulaczyk et al., 2001, 2000a]. Moreover, it has been observed that the base of the stopped Ice Stream C is supercooled by \(-0.35^\circ\text{C}\) below the pressure melting point [Kamb, 2001a]. Also at Ice Stream C, a thick debris-bearing basal zone (\(\sim20\) m) has been observed with borehole video camera [Carey et al., 2003, 2001; Kamb, 2001b]. Relatively high debris content can be inferred from many of these video recordings (e.g., Figure 2a). Thick layers of clean ice are, however, abundant as well. Nevertheless, several-meter-thick layers of clean segregation ice can also be produced by frost heaving under permafrost conditions (D. Lawson, personal communication, 2002). In our opinion, the most compelling observation supporting applicability of our approach to simulating freeze-on beneath Ice Stream C is the existence of undeformed debris layers in the basal ice found at the UpC camp (Figure 2b). These relatively evenly spaced debris bands, which were found in several drilling locations, are macroscopically similar to ice-sediment interlayering generated by frost heaving under permafrost conditions (Figure 2c).

[27] In the parameterization of the ice stream system, based on Tulaczyk et al. [2000b], even an infinitesimally small initial basal freezing leads to an increase in freeze-on rate because it triggers a positive feedback that leads to complete ice stream shutdown and, hence, to nil basal shear heating. The numerical values of the relevant parameters in our model have been chosen to emulate the well studied UpB and UpC areas on Whillans Ice Stream and Ice Stream C (Figure 1) [Engelhard and Kamb, 1997, 1998; Kamb, 1991; Tulaczyk et al., 2001, 2000a]. We have performed sensitivity studies, which show that the fundamental results of our model are not dependent upon the selection of these glaciological parameters. In the model runs presented here, the surface slope was set to \(\alpha = 0.0014\), the driving stress to \(\tau_d = \rho gh\) \(\sin \alpha = 12.6\) kPa, the ice thickness to \(H = 1000\) m, and the ice stream width to \(W = 36 \times 10^3\) m. The shear strength of the till column is derived from the Mohr-Coulomb criterion:

\[\tau_f = c + p'\tan \phi\]  

(12)

where \(c\) is the cohesion of the till, \(\phi\) the angle of friction. The basal shear stress, \(\tau_b\), is related to the basal shear strength, \(\tau_f\), or the driving stress, \(\tau_d\), through the following criteria [Tulaczyk et al., 2000a]:

\[\begin{align*}
\tau_b &= \tau_f & \text{if } \tau_f < \tau_d \\
\tau_b &= \tau_d & \text{if } \tau_f \geq \tau_d
\end{align*}\]  

(13)

[28] The effective pressure is \(p' = p_n - p_w = p_n - p_b - u\). The surface velocity of the ice stream, \(U_s\), is calculated from [Raymond, 1996, equation (38); Tulaczyk et al., 2000b, equation (15)]:

\[U_s = U_b + U_{def} = \left[1 - \left(\frac{\tau_b}{\tau_d}\right)^n\left(\frac{W}{2H}\right)^{n+1} + \left(\frac{\tau_b}{\tau_d}\right)^n\right]U_d\]  

(14)

where \(U_b\) is the basal velocity component and \(U_{def}\) is the velocity component of due to internal deformation, \(W\) is ice stream width, \(H\) is ice thickness, while \(U_d = 2\tau_d^0H(n + 1)^{-1}B^{-n}\) is the surface velocity for ice moving purely by internal deformation with \(\tau_b = \tau_d\) \((n \text{ and } B \text{ are ice flow-law constants})\).

3.3. Transport Equations and Boundary Conditions

[29] The three fundamental variables of our model \((u, T, C)\) are calculated from three partial differential transport equations (equations (5), (6), and (7)). Each transport equation is constrained by appropriate lower and upper boundary conditions. The lower boundary at the till base constitutes the ‘warm’ end of the system, which is associated with relatively simple balance equations. Here, geothermal heat flux enters the till and the thermal boundary condition is \(\partial T/\partial z = G/K\). We assume no change in solute concentration across the lower boundary, so \(\partial C/\partial z = 0\). As mentioned previously, we operate with two cases of subglacial water systems. In the closed-system case there is no flux of water across the lower boundary, and \(\partial u/\partial z = 0\). In the open-system a gradient in excess water pressure can withdraw water from the sub-till sediment. These two cases are comparable to the hydrogeological cases of constant gradient and constant head. The upper boundary constitutes the ‘cold’ end of the system. The heat budget at the upper boundary has four components related to entry of heat from...
the till below, exit of heat into the overlying ice, frictional heat from basal sliding, and latent heat of fusion from ice-water phase transition. The rate of melting or freezing, \( \dot{m} \), is determined from the heat budget, which changes during the stoppage of the ice stream. The heat budget of the upper boundary is thus:

\[
\frac{\partial T}{\partial z} K_i - \dot{b}_i \dot{K}_i + \tau_b U_b - \rho_i \dot{m} L = 0
\]

where \( T \) is temperature in till, \( z \) is depth coordinate, \( K_i \) is coefficient of thermal conductivity of till, \( \dot{b}_i \) is basal temperature gradient of ice, \( \dot{K}_i \) is coefficient of thermal conductivity of ice, \( \tau_b \) is basal shear stress, \( U_b \) is basal velocity, \( \rho_i \) is density of water, \( \dot{m} \) is melting rate (<0 for freezing), and \( L \) is the coefficient for latent heat of fusion. Also in accordance with permafrost observations, we assume rejection of solutes in the water-ice phase change [Panday and Corapcioglu, 1991]. Hence, there is no transport of solutes across the upper boundary. Excess water pressure at the freezing ice interface is calculated from the Clapeyron equation (2), under the assumption that force balance within the basal ice and ice lenses prescribes an ice pressure, \( p_i \), equal to the gravitational overburden pressure, \( p_o \) (see Figure 5). Rearranging equation (2) under this assumption and using the excess water pressure definition, \( u = p_w - p_i \), yields an upper boundary for \( u \):

\[
u_{iw} = \left( \frac{p_w}{\rho_i} \right) p_o + \left( \frac{L p_o}{273.15} \right) T + p_o - p_i
\]

where subscript ‘\( iw \)’ refers to the ice-water interface. This equation introduces full coupling of the three fundamental model variables \( u, T, \) and \( z \).

The upper boundary of our model is further complicated by ice lens development. The upper boundary thus moves down through the till column in discrete steps. The new location of the upper boundary is thus prescribed at the node, which meets the ice lens criterion (equation (9)). As in general frost heave models we only treat the flow toward the upper boundary, which is located at the base of the youngest ice lens. Secondary flows of heat, water, and solutes within isolated till layers above the upper boundary (see Figure 4) are not treated explicitly.

Regelation comprises the final complication of the upper boundary condition. An increase in effective stress at the supercooling upper boundary may trigger regelation of ice into till pore spaces (equation (10)). We assume that intrusion of regelation ice into till pores will act to increase pore water pressure because regelation ice is taking up space previously occupied by pore water. This self-regulatory trend of regelation is discussed by Alley et al. [1997]. The upper boundary thus contains two opposing processes. Phase equilibrium of the ice segregation process reduces the water pressure in response to basal cooling, but the associated increase in effective stress may trigger regelation, which increases the water pressure following an empirical formulation for a swelling till [Tulaczyk et al., 2000a].

### 3.4. Initial Conditions

Till properties and initial conditions used in our model calculations are generally based on results of borehole observations made on several West Antarctic ice streams [Engelhardt et al., 1990; Engelhardt and Kamb, 1997; Engelhardt and Kamb, 1998; Kamb, 1991, 2001; Tulaczyk et al., 2001, 2000a]. These observations have been discussed previously, so that we will here simply list the values of till properties and the generalized ice stream characteristics. Values of till parameters are listed in Table 1, and values of ice parameters are listed in Table 2.

The geothermal heat flux is based on modeling of basal heat flow at Siple Dome (H. Engelhardt, California Institute of Technology, personal communication, 2002) and the conductivity coefficient of till to ice is based on the experimental results of Iverson and Semmens [1995]. There is no flow in the system when we initiate the numerical simulations. We induce freezing at the upper boundary of the till column by an infinitesimal increase in basal temperature gradient (~0.001°C m⁻¹) above the critical value (~0.053°C m⁻¹) that brings the system into freezing mode [Tulaczyk et al., 2000b]. The initial properties of time-dependent parameters are listed in Table 3.

### 4. Results

The final output of our till freeze-on model can be expressed in terms of physical changes of till properties (e.g., porosity), compositional changes of pore water and segregation ice (e.g., solute concentration or isotopic composition), and also ice lens formation through time. As previously addressed, we operate with two cases of hydro-geological setting, i.e. a closed water system (case 1) and an open water system (case 2). We treat these very different settings in order to explore the effects of water availability upon the physical changes in the basal zone, which may be related to ice stream stoppage through increased basal drag. We also explore the sensitivity of our model to changes in

### Table 1. Symbols and Values of Constant Properties for the Numerical Till Column

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>( 2.0 \times 10^3 ) Pa</td>
<td>cohesion</td>
</tr>
<tr>
<td>( c_v )</td>
<td>( 10^{-3} ) m² s⁻¹</td>
<td>hydraulic diffusion coefficient</td>
</tr>
<tr>
<td>( G )</td>
<td>( 70 \times 10^{-3} ) J s⁻¹ m⁻²</td>
<td>geothermal heat flux</td>
</tr>
<tr>
<td>( K_i )</td>
<td>( 10^{-11} ) m² s⁻¹</td>
<td>hydraulic conductivity coefficient of till to ice</td>
</tr>
<tr>
<td>( \rho_{iw} )</td>
<td>( 25 - 100 \times 10^9 ) Pa</td>
<td>ice-water interfacial pressure</td>
</tr>
<tr>
<td>( \sigma_{iw} )</td>
<td>( 3.4 \times 10^{-4} ) J m⁻²</td>
<td>specific surface energy of ice-water interface</td>
</tr>
<tr>
<td>( \nu_w )</td>
<td>( 10^{-10} ) m² s⁻¹</td>
<td>chemical diffusion coefficient</td>
</tr>
<tr>
<td>( \nu_v )</td>
<td>( 7.6 \times 10^{-13} ) m² s⁻¹</td>
<td>thermal diffusion coefficient</td>
</tr>
<tr>
<td>( \rho_i )</td>
<td>( 2600 ) kg m⁻³</td>
<td>density of solid till particles</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>( 1000 ) kg m⁻³</td>
<td>density of pore water</td>
</tr>
<tr>
<td>( \phi )</td>
<td>( 22^\circ )</td>
<td>angle of friction</td>
</tr>
</tbody>
</table>
Previous work suggested that the fine-grained nature of the clay-rich UpB till sampled from beneath the Whillans Ice Stream is associated with an interfacial pressure of at least 100 kPa, whereas the coarse-grained till from Breidamerkurjökull in Iceland is associated with an interfacial pressure of just a few kPa [Tulaczyk, 1999]. We thus compare the model results using \( p_{inw} = 100 \) kPa (high surface tension) and \( p_{inw} = 25 \) kPa (low surface tension). The results presented for each of the two hydrogeological settings (case 1 and case 2) thus contain two model runs (low surface tension till of 25 kPa and high surface tension till of 100 kPa). Whereas the model results are displayed together graphically, the hydrogeological case results are described separately in the following sections.

### 4.1. Case 1: Closed Water System

When the substratum below the till is impermeable, basal freeze-on is fed purely by extraction of pore water that was initially present in the till layer. With our model, we are able to simulate the progression of a freezing front that advances through the till domain. However, numerical instability arises when the till column below the freezing front is reduced to less than 10–20% of the original thickness. The calculations are therefore terminated somewhat prior to complete freeze-up. Freeze-induced changes of the most significant parameters of the basal regimen are seen in Figure 7. The till consolidates as freeze-on extracts and consumes pore water. As a result of this consolidation process, the shear strength of the till increases while the ice velocity drops (Figure 7a). When the till strength reaches the driving stress, the simulated ice stream system shuts down completely. In the closed-system case this happens after 65 years of freezing irrespective of the assumed till surface tension. Enhanced ice flow is prevented from then on and the ice velocity is reduced to the level of internal deformation. The corresponding changes in freezing rate and till shear strength are shown in Figures 7b and 7c.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Initial Value/Unit</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>([3.0 3.0]) %</td>
<td>solute concentration in till</td>
</tr>
<tr>
<td>( m )</td>
<td>0 m s(^{-1})</td>
<td>melting rate/freezing rate</td>
</tr>
<tr>
<td>( T )</td>
<td>([-0.7 -0.5]) °C</td>
<td>temperature distribution in till</td>
</tr>
<tr>
<td>( u )</td>
<td>([-2.0 -2.0]) (\times) 10(^3) Pa</td>
<td>excess water pressure distribution</td>
</tr>
<tr>
<td>( U_b )</td>
<td>450 m s(^{-1})</td>
<td>ice stream velocity</td>
</tr>
<tr>
<td>( z )</td>
<td>([0.0 5.0]) m</td>
<td>depth below ice base</td>
</tr>
<tr>
<td>( \phi )</td>
<td>([40 40])%</td>
<td>porosity distribution in till</td>
</tr>
<tr>
<td>( \nu_v )</td>
<td>([0.0]) m s(^{-1})</td>
<td>vertical water flow velocity in till</td>
</tr>
<tr>
<td>( \tau_b )</td>
<td>(2.7 \times 10^3) Pa</td>
<td>basal shear stress at ice-till interface</td>
</tr>
<tr>
<td>( \tau_f )</td>
<td>([2.7 2.0]) (\times) 10(^3) Pa</td>
<td>shear strength of till</td>
</tr>
</tbody>
</table>

*Depth-dependent parameters are listed \([x y]\), where \(x\) refers to the upper value at the ice-till interface, and \(y\) is the lower value at the base of the till.*

Figure 7. Changes in ice stream velocity, melting rate, and basal shear strength with time. Closed water system conditions result in: (a) decrease in ice velocity caused by increased basal shear strength, (b) decrease in melting rate (increase in freeze rate) caused by loss of frictional heat, and (c) increase in basal shear strength due to extraction of pore water. Open water system conditions result in: (d) decrease in ice velocity, (e) increase in freeze rate, and (f) increase in basal shear strength. Ice stream stoppage occurs when the basal shear strength \( (\tau_f) \) exceeds the driving stress \( (\tau_d) \). Arrows indicate trend of till strength development subsequent to ice lens initiation.
The evolution of the coupled ice-till system shown in Figures 7a–7c is gradual during the first 50 model years. These changes accelerate thereafter due to the non-linearly decreasing shear heating. The basal system equilibrates after stagnation at 65 model years with latent heat of freezing replacing shear heating in the basal heat budget. The freeze rate stabilizes at 3.7 mm a\(^{-1}\) (Figure 7b) and it remains constant after stagnation. Ice stream force equilibrium is maintained purely by basal drag after shut down, so the basal shear stress is truncated by the driving stress. The shear strength of the till increases further due to the continuing extraction of pore water during growth of segregation ice (Figure 7c).

Changes of till porosity induced by basal freeze-on are shown in the depth-time diagrams presented in Figure 8. The porosity changes are relatively small during ice stream slow down; it drops from 40% to just 39%. Large porosity changes are almost entirely associated with the stagnant stage.

In the closed water system, it takes 120 years for the freezing front to reach a depth of 4.14 m when the surface tension of the till is 25 kPa (Figure 8a). During the first 64 years 0.046 m of clean segregation ice accretes onto the ice base. Thereafter, the freezing front moves into the till and 17 ice lenses develop in 56 years. The lens thickness decreases with depth from 0.024 m to 0.011 m, while the lens spacing decreases from 0.31 m to 0.22 m. The minimum till porosity is reached deepest in the till where pore water is extracted throughout the simulation time. The porosity of the lowermost till decreases from 40% to 32% when the surface tension is 25 kPa, and the associated increase in shear strength is from 2.7 kPa to 49 kPa (Figure 8a).

When the till has a surface tension of 100 kPa it takes 230 model years for the freeze-front to reach a depth of 4.17 m when the water system is closed (Figure 8b). Accretion of clean ice onto the ice base amounts to 0.26 m before the first ice lens is initiated at 0.83 m after 120 years of freezing. In this till case only 6 ice lenses develop in the till. The thickness of the first 5 lenses decreases with depth from 0.18 m to 0.05 m, and the final active lens is 0.023 m thick when numerical instability terminated the simulation. The lens spacing is initially 0.83 m but it decreases to 0.56 m with depth. Till porosity is reduced significantly when surface tension is high. For till with surface tension of 100 kPa, the porosity in the lowermost till decreases from 40% to less than 25% after 230 years of freezing. The shear strength of the lowermost till increases significantly from 20 kPa to 120 kPa as shown in Figure 8b.

The till becomes supercooled due to the combined influence of surface tension and the solutes present in the till. If the surface tension is low (25 kPa), ice lenses are initiated when the temperature at the ice base reaches \(-0.89^\circ\)C. This corresponds to a depression of the freezing point by approximately \(-0.23^\circ\)C below the pressure-melting point, which is \(-0.66^\circ\)C for air-free water with \(p_n = 9.2\) MPa [Paterson, 1994, p. 212]. When the surface tension is high (100 kPa),
Ice lenses are initiated when the temperature at the ice base reaches \(-1.0^\circ C\). This temperature corresponds to a depression of the freezing point by \(-0.34^\circ C\) from pressure-melting point. A similar freezing-point depression is observed beneath the stopped Ice Stream C [Kamb, 2001a]. Temperatures are listed in the time-depth images of Figure 9 displaying the evolution of solute concentration in the freezing system. The pore sizes of tills dictate the level of freezing point depression, and the supercooling effect should be most pronounced in fine-grained sediments with high surface tension effects. When the sediment is very fine-grained, our model requires considerable simulation time to develop the internal ice pressure that is required to initiate a new lens. When the next lens finally forms it is located at a greater depth than would be the case for a till with low surface tension. In general, low surface tension is associated with thin and closely spaced ice lenses, whereas high surface tension till is associated with fewer but significantly thicker ice lenses that are also more widely spaced.

Solute concentrations are rejected from liquid water that freezes and they accumulate below the freezing interface. The longer the front remains stationary, the higher is the solute concentration immediately below. Although diffusion redistributes solutes according to the concentration gradient (downward), the advection component of the transport (upward) is more effective. Accumulation of solutes near the freezing ice base is shown in Figure 9, where the evolution of solute concentration is displayed in depth-time diagrams (analogous to the porosity diagrams in Figure 8).

For till with a low surface tension in the closed water system, the assumed initial solute concentration of 3% rises to peak values of 3.6–3.8% beneath ice lenses (Figure 9a). In the high surface tension case, the peak values are 4.3–5.6% (Figure 9b). The concentration is higher in the latter case because the freezing front progresses more slowly through the till. When the active interface moves into the till, we do not calculate the subsequent diffusion of solutes in the isolated till layers trapped between ice lenses above the upper boundary. Diffusion should, however, eliminate solute concentration gradients in these isolated layers.

When the freeze-front is located at the original ice base, the critical water pressure (equation (10)) is not reached and regelation is not triggered because the condition for ice lensing (equation (9)) is reached sooner. Regelation is in our model triggered mainly by a jump in effective pressure associated with ice lens initiation. Conditions in the simulated sub-ice stream environment do not favor regelation. The model predicts ice intrusion depths that are restricted to a few mm’s or few cm’s. The average intrusion depth of ice by regelation is 0.035 m in the case of low surface tension and 0.0076 m in the case of high surface tension.

4.2. Case 2: Open Water System

In the open system case, we assume that water is available for withdrawal from a sub-till aquifer. The rate of groundwater input into the till is dictated by the basal water pressure gradient in the till. The general results of the open
system case are similar to the results obtained for a closed water system. The pattern of ice lens development resembles the previous closed-system case, but there are differences in predicted lens thickness and spacing. In spite of our initial expectation that opening the till to water influx from below would hinder ice stream slowdown, the period of predicted ice stream activity is only 8 years longer than in the closed system case (Figure 7d). The freeze rate stabilizes at 3.6 mm a\(^{-1}\) (Figure 7e), which is also similar to the value obtained in a closed water system. The changes in till properties are as expected considerably different in the two water system cases. In the closed system case, the basal shear strength increased progressively (Figure 7c) whereas it levels out in the open case (Figure 7f).

The till shear strength increases to just 17 kPa when the surface tension is low (25 kPa) and the water system is open (Figure 8c). In the closed system case, the corresponding result was 49 kPa (Figure 8a). For till with a high level of surface tension this difference is even more pronounced. The basal shear strength reaches only 23 kPa in an open water system (Figure 8d) while it attained 120 kPa in a closed water system (Figure 8b). As in case 1, the bulk porosity changes also occur after complete shutdown, which occurs at 73 model years in this case. Importantly, porosity changes cease after 100 model years because the extraction of water by freezing is balanced by the influx of groundwater across the lower boundary. Significant porosity changes are thus restricted to the limited period between 73 and 100 model years. In a closed water system, the porosity drops continuously due to continuous extraction of pore water, which is consumed by the freeze-on process (Figures 8a, 8b, and 7c).

Ice lenses develop in a spatial pattern that resembles the closed system case. The uppermost lenses are thicker than the lower ones. The distance between individual lenses decreases with depth, which was also the case in the previous section. The open water system enables lenses to attain a greater thickness, especially in the case of high surface tension till (100 kPa) where lenses measure 0.25–0.28 m. The spacing between these lenses is 0.84–0.85 m. For till with a surface tension of 25 kPa the uppermost lens thickness is 0.046 m but it decreases to 0.011 m with depth. The associated lens spacing is 0.21–0.31 m, decreasing with depth.

For till with a surface tension of 25 kPa, the temperature at the ice-till interface reaches \(-0.89^\circ C\) when the first lens is initiated at 74 model years. For till with 100 kPa surface tension the first lens occurs after 136 years when the temperature is \(-1.0^\circ C\) (see Figure 9). These temperatures are identical to the values obtained in the closed system case, although the timing of ice lensing is offset by 10 years and 16 years in the respective cases of low and high surface tension.

Concentration of solutes below ice lenses is slightly higher in the open-system case (Figures 9c and 9d) compared with the closed-system case (Figures 9a and 9b). These increased values are due to a more stationary freeze-front that moves more slowly through the till because water is more abundant in this system. When the surface tension in the till is 25 kPa the peak levels of concentration are 3.4–3.8\%o beneath ice lenses. When the surface tension is 100 kPa the corresponding values are 5.4–5.6\%o.

A thin veneer of regelation ice intrudes the pore spaces beneath ice lenses that develop within the till. The average penetration depth of ice is 0.0090 m for low surface tension (25 kPa) and 0.050 m for high surface tension (100 kPa).

5. Comparison to Observations

Many of our model parameters have been tuned to resemble the sub-ice stream environment of the West Antarctic ice sheet, which represents the best studied sub-glacial zone of modern ice sheets. The subglacial sediment in the Ross Sea sector is a clay-rich till with a surface tension estimated to be ca. 104 kPa [Tulaczyk, 1999]. This number is most likely an underestimate because it is based on the assumption that all particles are spherical in shape. With more realistic assumption about the shape of particles, particularly clays, this estimate may increase even by an order of magnitude. In general, our input data reflect the particularly well-studied UpB area of the fast flowing Whillans Ice Stream. However, we compare model output mostly with subglacial observations from the UpC area of the recently stopped Ice Stream C because this site provides the best analog to our model simulations.

Our results show that it is feasible for freeze-on to increase the shear strength of initially weak and porous till (~3 kPa), not only to a level that prevents basal sliding and ice streaming (~10–20 kPa), but also to level that produces a relatively high degree of consolidation (>100 kPa). The predictions also show that the till beneath a recently stopped ice stream may exhibit porosity that is only a few percent lower than till porosity associated with active ice streaming. Thus, the till can remain largely unfrozen as observed in recent radar reflection surveys from the recently stopped Ice Stream C [Bentley et al., 1998; Gades et al., 2000]. Till subjected to prolonged periods of basal freeze-on (100s of years) can attain very high levels of consolidation and shear strength [Christofferson and Tulaczyk, 2003]. This freeze-on-driven mechanism of till consolidation may be responsible for strongly consolidated till layers present at the bottom of the Ross Sea over which the West Antarctic ice sheet advanced during the last glacial maximum [Anderson, 1999, p. 102]. A recent seismic study in the western Ross Sea shows a distinct change in basal character between ice advance leading to the last Glacial Maximum and subsequent recession. Lateral transport of basal debris stopped, possibly as a result of onset of basal freezing [Howat and Domack, 2003].

Kamb [2001a] reports several distinct subglacial features at the bed of Ice Stream C. The basal temperature gradient is 0.054°C m\(^{-1}\) with an estimated freeze-on rate of 4.5 mm a\(^{-1}\). The basal temperature is \(-1.1^\circ C\), which is \(-0.35^\circ C\) below the pressure-melting point (e.g. \(-0.71^\circ C\) for ice 1057 m thick, ibid., p. 160). Yet, the underlying till is unfrozen. Outside the ice stream (beneath interstream ridges), the basal temperature is \(-0.6^\circ C\) to \(-2.7^\circ C\) below the pressure melting point and there, the ice base is assumed to be frozen to the bed. In our model, the simulated basal temperatures are depressed by several tenths of a degree below the pressure melting point. This supercooling arises from the combined effects of ice-water surface tension and the presence of solutes. We obtain a basal temperature of
−1.0°C at the ice-till interface after 120 years of basal freezing. This temperature is approximately −0.3°C below the pressure melting point and the corresponding freeze-on rate is ~3.7 mm a⁻¹. Thus, our model simulations reproduce several important observations from beneath Ice Stream C. [53] Model output also reproduces a common feature observed in the basal zones of deep ice cores recovered from polar ice sheets [Gow et al., 1979; Herron and Langway, 1979; Koerner and Fisher, 1979]. The model yields a layered basal ice facies that consists of inter-changing bands of clean ice (segregation ice lenses) and dirty ice (frozen-on till). Similar basal layers have recently been observed in boreholes drilled to the bottom of Ice Stream C [Carsey et al., 2003; Kanh, 2001b]. As seen in Figures 2a and 2b, borehole video recordings show a debris-bearing basal ice layer, which is similar in character to our model predictions shown in Figures 8 and 9. The basal layer has a predominance of ice over debris. In comparison, our model overestimates the debris content. Typical frozen-on basal layers tend to have a volumetric debris content of ~10% or less [Kirkbride, 1995]. Decimeter-sized debris layers are, however, found in the basal zone of the Byrd ice core [Gow et al., 1979] and the Camp Century ice core in Greenland [Herron and Langway, 1979]. Our model based on frost heave physics predicts a volumetric debris content in the basal ice of ~40% or more. This error suggests that our treatment of the freezing ice base does not include all of the necessary physics. Supply of additional water from a widespread, throughgoing basal water system, not included in our model, could account for this discrepancy between model results and observations. Alternatively, we may be underestimating the rate of water transport toward a growing ice base (e.g., too low hydraulic conductivity) or setting the till surface tension parameter too low. Further laboratory and/or borehole constraints may be necessary to resolve these questions. [53] In spite of some model shortcomings, we are in general satisfied with model performance because we can show that basal freeze-on can produce characteristic stratified debris rich basal ice layers, which have been observed beneath ice sheets [Alley et al., 1997; Boulton and Spring, 1986; Gow et al., 1979, 1997; Herron and Langway, 1979; Knight, 1997; Koerner and Fisher, 1979], as well as many alpine glaciers [Hubbard, 1991; Hubbard and Sharp, 1995; Lawson and Kulla, 1977; Lawson et al., 1998]. When viewed on a greater scale, basal freeze-on provides a mechanism that switches off bed lubrication in response to ice stream over-thinning. This negative feedback effect may have significant control over ice sheet mass balance [Joughin and Tulaczyk, 2002].

6. Conclusions

[56] In a series of numerical experiments we have investigated the response of fine-grained subglacial till to basal freeze-on triggered beneath a fast flowing ice stream. The extraction of pore water, which is associated with ice segregation growth, consolidates the till and increases the basal shear strength. The ice stream slows down due to increased basal drag and loss of frictional heat. Complete stoppage is gained after 60–75 years of freezing because the basal shear strength reaches the driving stress (~13 kPa). This period is, however, associated only with minor changes in the physical properties of the till. The porosity changes from 40% to just 39%, and this may explain why geo-physical profiles from Ice Stream C show a wet and porous bed. Significant porosity changes are in our model associated only with prolonged periods of freezing (>200 years). If the till is fine-grained, porosities can reach low values (~25%), and the till strength subsequently high values (~120 kPa). However, this is only the case if the substratum beneath the till is impermeable (‘closed system’). Water inflow from sediments beneath the till (‘open system’) impedes till consolidation significantly because the basal freeze-on does not have to consume till pore water alone. In this case the porosity reduction is truncated at ca. 35% and the basal shear strength at ca. 20 kPa.

[57] Our model reproduces a debris-bearing basal ice with layered structure. Such structure has been observed in numerous deep ice cores that reached the ice base of modern ice sheets. In our model, this ice-debris interlayering results from development of segregation ice lenses within the subglacial till. Fine-grained tills with high surface tension should be associated with relatively thick ice lenses (~0.1–0.25 m) and relatively wide spacing (~0.5–0.8 m). A coarser grained till with low surface tension yields thinner but more abundant ice lenses (~0.01–0.05 m) that are also more closely spaced (~0.2–0.3 m). The model has a tendency to overestimate the debris content of basal ice (~40% by volume) when compared to measurements from basal zones of glaciers and ice sheets (~10%). We expect that this error is a reflection of the upper boundary not quite capturing the precise physics of the downward progressing freezing front. We are, however, encouraged by the bed conditions predicted by the model. Ice stream stoppage is associated with a highly porous, unfrozen bed that is supercooled by ca. −0.35°C below the pressure melting point. This is a fairly accurate depiction of the bed properties of Ice Stream C. This detailed study suggests that basal freeze-on is a powerful subglacial mechanism with major implications for ice dynamics and for development of physical properties of subglacial sediments.

Notation

Model variables

\( C \) solute concentration, \(^\%\).
\( T \) temperature, \(^\circ\text{C}\).
\( U \) excess pore water pressure, \(\text{Pa} \).
\( t \) time, \(s\).
\( z \) depth in till below ice base, \(m\).

Ice parameters

\( m \) melting rate, \(m \, s^{-1} (<0 \text{ for freezing})\).
\( H \) ice thickness, \(m\).
$K_t$ thermal conductivity for ice, J m$^{-1}$ s$^{-1}$ K$^{-1}$.
$L$ coefficient for latent heat of fusion, J kg$^{-1}$.
$U_b$ basal velocity, m s$^{-1}$.
$U_s$ surface velocity, m s$^{-1}$.
$W$ ice stream width, m.
$\alpha$ surface slope of ice stream.
$\theta_b$ temperature gradient of basal ice, K m$^{-1}$.
$\rho_i$ density of ice, kg m$^{-3}$.
$\sigma_{iw}$ ice-water surface energy, J m$^{-2}$.
$\tau_d$ driving stress, Pa.

Till parameters

- $C$: cohesion of till, Pa.
- $c_s$: hydraulic diffusion coefficient, m$^2$s$^{-1}$.
- $dA/dV$: curvature of ice-water interface, m$^{-1}$.
- $g$: constant of gravity, m s$^{-2}$.
- $G$: geothermal flux, J m$^{-2}$ s$^{-1}$.
- $K_h$: hydraulic conductivity, m s$^{-1}$.
- $K_r$: thermal conductivity, J m$^{-1}$ s$^{-1}$ K$^{-1}$.
- $K_{reg}$: regelation conductivity, m$^2$ Pa$^{-1}$ s$^{-1}$.
- $r_p$: characteristic particle radius, m.
- $u_w$: flow of water, m s$^{-1}$.
- $S_{SM}$: specific surface area, m$^{-1}$.
- $V_r$: regelation velocity, m s$^{-1}$.
- $z_r$: ice penetration depth by regelation, m.
- $\phi$: porosity, $\%$.
- $\phi_i$: angle of internal friction, $\circ$.
- $w$: chemical diffusion coefficient, m$^2$s$^{-1}$.
- $K_w$: thermal diffusion coefficient, m$^2$s$^{-1}$.
- $\rho_w$: density of water, kg m$^{-3}$.
- $\tau_b$: basal shear stress, Pa.
- $\sigma_{bas}$: basal shear strength, Pa.

Pressure components

- $P_e$: effective pressure, Pa.
- $p_i$: ice pressure, Pa.
- $p_g$: total gravitational pressure, Pa.
- $p_w$: pore water pressure, Pa.
- $p_{w,crit}$: critical pore water pressure for regelation, Pa.
- $p_h$: hydrostatic pore water pressure, Pa.
- $p_o$: osmotic pressure, Pa.
- $p_{iw}$: ice-water interfacial pressure, Pa.
- $\Delta p$: regulation driving stress, Pa.
- $u_{d,p}$: excess pore water pressure at freezing, Pa.

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