Signature of palæo-ice-stream stagnation: till consolidation induced by basal freeze-on

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Ice streams affect behaviour and stability of ice sheets (Alley et al. 1987). Fast ice-stream flow arises from the lubricating presence of weak and deformable till (Blankenship et al. 1987; Engelhardt & Kamb 1997; Kamb 1991; Tulaczyk et al. 2000a). This type of ice-bed lubrication occurs when the frictional resistance of subglacial sediments becomes very small (~1–2 kPa; Kamb 2001) due to high subglacial water pressures that are close to the flotation level of the ice sheet (Engelhardt & Kamb 1997; Tulaczyk et al. 2001). Changes in water content and water pressure influence the strength and deformational behaviour of subglacial till (Tulaczyk et al. 2000a). Ice-stream dynamics are thus sensitive to changes in the basal melting/freezing rate (Tulaczyk et al. 2000b). For instance, a switch from basal melting to basal freezing may have caused the stagnation of Ice Stream C in West Antarctica approximately 150 years ago (Price et al. 2001; Christoffersen & Tulaczyk in press (a); Bougamont et al. in press). Such major perturbations in ice stream flow may control the mass balance of ice sheets (Joughin & Tulaczyk 2002). It is therefore important to investigate the potential effect of basal freezing on glacier dynamics in order to reconstruct the history of ice-sheet interactions with the climate system and global sea level.

Till deposited by former ice sheets is a potential source of important palæo-glaciological information. Physical properties of tills are to a great extent shaped by subglacial effective pressure (Boulton & Dobbie 1993). A number of studies have utilized geotechnical concepts to estimate maximum past effective pressure beneath ice streams and ice sheets (Harrison 1958; Larsen et al. 1995; Sauer & Christiansen 1988; Sauer et al. 1993; Tulaczyk et al. 2001). Theoretical investigations of subglacial conditions have previously focused on the response of subglacial water pressure to basal melting and drainage into aquifer systems (Boulton & Dobbie 1993; Piotrowski & Kraus 1997). Impact of freezing on subglacial processes has been investigated to a lesser extent, mostly just in the context of water drainage beneath a melting-based ice sheet overriding permafrozen ground (Cutler et al. 2000; Mickelson et al. 1983). Basal freeze-on is a very different subglacial condition that includes interactions between a freezing ice base and unfrozen subglacial sediments. Theoretical investigations of freezing ice beds are rare and the physical response of subglacial sediments to basal freeze-on is uncertain. Yet, the recent stoppage of Ice Stream C is a strong indicator of the significant influence that basal freeze-on may have on ice-stream dynamics and ice-sheet configuration (Bougamont et al. in press). Criteria for identifying palæo-ice streams and their dynamics should therefore be an integral part of palæo-ice-sheet reconstruction (Stokes & Clark 1999, 2001; Boulton et al. 2001).

The two most prominent ice streams that appear to...
have affected the southwest edge of the Fennoscandian ice sheet are shown in Fig. 1A. The Norwegian Channel Ice Stream flowed around the southern tip of Norway (Sejrup et al. 1998; Sejrup et al. 2003) and the Baltic Ice Stream flowed in the Baltic Sea basin (Boulton et al. 2001; Houmark-Nielsen 1987; Punkari 1995; Jørgensen & Piotrowski 2003). Here, we argue that unusual till properties observed at several Scandinavian sites may have resulted from basal freezing beneath palaeo-ice streams. Numerical models can be used to simulate changes of till properties during ice-stream stagnation caused by a transition from basal melting to basal freezing. Despite a relatively mild palaeo-climate, we show that the base of mid-latitude palaeo-ice streams may have experienced periods of basal freezing due to horizontal advection of cold ice.

Geologic observations

Our interest in the possible role of basal freezing in controlling till consolidation was sparked by some characteristic features of till deposited by the Baltic Ice Stream. The ice stream was a prominent and highly dynamic feature of the Fennoscandian ice sheet during the final Pleistocene glacial advance (Punkari 1995; Punkari 1997; Boulton et al. 2001). It advanced Denmark from SE approximately 15 ka BP (Houmark-Nielsen 1987). The maximum extent is outlined in Fig. 1A, which also shows the maximum extent of the main NE ice advance, c. 18–22 ka BP. Till deposits in Storebælt (location ‘S’ in Fig. 1B) have been studied extensively for several decades due to construction of the Fixed Link: a bridge/tunnel combination between Sjælland (Sealand island) and Fyn (Funen island). The Fixed Link is so far the largest construction in Denmark and it consists of an 8-km bored tunnel, a 6-km low bridge and a 6-km elevated suspension bridge with pylons exceeding 250 m in height. Physical properties of the Quaternary till cover were investigated through an extensive drilling programme prior to construction (e.g. Larsen et al. 1982). Cone penetrometer tests and vane shear strength measurements have revealed that the uppermost till unit, the Sprogø Till, exhibits unusual yet regionally consistent strength profiles (Danish Geotechnical Institute, 1990, unpublished geotechnical report). The uppermost part of Sprogø Till is often found to be well consolidated and strong, while the lower part is poorly consolidated and weak (Foged et al. 1995). A generalized geotechnical profile of till strength variations with depth is shown in Fig. 2. Drilling programmes in Storebælt have shown that the Sprogø Till is an integral part of a large marginal moraine from the Baltic Ice Stream advance (Foged et al. 1995). The vertical extent of the Sprogø Till is around 10 m (Foged et al. 1995). Although this is a somewhat large thickness when compared to non-ice-stream till deposits (e.g. Houmark-Nielsen 1987), it is similar to the thickness of sub-ice-stream till layers observed in West Antarctica (Blankenship et al. 1987; Alley et al. 1987; Tulaczyk et al. 2001).

In a typical hydrogeologic setting, shear strength of
Subglacial sediments is expected to increase with depth due to an increase of effective pressure with depth (Boulton & Dobbie 1993). This is obviously not the case in the till discussed here, where strong till overlies considerably weaker till. Several different geological mechanisms have been proposed for the unusual till properties. Examples thereof are glaciotectonic thrusting, subglacially buried ice, gas seepage and postglacial alteration processes (Danish Geotechnical Institute, 1990, unpublished geotechnical report). Most of these mechanisms are qualitative propositions and so far a quantitative portrayal of the origin of the anomalous till properties remains to be done. A yet to be explored mechanism, which may in a simple way explain the till property variation with depth, is the effect of a freezing ice-stream base.

Till with properties similar to those of Sprogø Till in Storebælt has been found near the township of Flakkebjerg, which is situated about 10 km NE of the subglacial sediments is expected to increase with depth due to an increase of effective pressure with depth (Boulton & Dobbie 1993). This is obviously not the case in the till discussed here, where strong till overlies considerably weaker till. Several different geological mechanisms have been proposed for the unusual till properties. Examples thereof are glaciotectonic thrusting, subglacially buried ice, gas seepage and postglacial alteration processes (Danish Geotechnical Institute, 1990, unpublished geotechnical report). Most of these mechanisms are qualitative propositions and so far a quantitative portrayal of the origin of the anomalous till properties remains to be done. A yet to be explored mechanism, which may in a simple way explain the till property variation with depth, is the effect of a freezing ice-stream base.

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**Fig. 2.** Generalized shear strength profile from Storebælt (location ‘S’ in Fig. 1B) showing characteristic bulge-shaped variation of till strength with depth (reproduced after Danish Geotechnical Institute, 1990, unpublished geotechnical report).

**Fig. 3.** Geology and geotechnical properties observed at Flakkebjerg (location ‘F’ in Fig. 1B): A. Lithological log showing lodgement till overlain by proglacial recessive sediments deposited during glacial decay (modified after Klint & Gravesen 1999). The three photographs noted are seen in Fig. 4A–C. B. Clast fabric orientations in lateglacial diamicton, upper hard till and lower soft till (lower hemisphere, Schmidt equal area projection). C. Fracture orientations in layers according to (B) (lower hemisphere, Schmidt equal area projection). D. Grain-size distribution of upper hard till (solid line) and lower soft till (dashed line). E. Variation in shear strength with depth based on torvane measurements made in the till (data from Christoffersen (1998)).
existing Storebælt shoreline (location ‘F’ in Fig. 1B). Here, several pits were excavated to depths of 5 m (Klint & Gravesen 1999; Christoffersen 1998).

The geologic sequence exposed at Flakkebjerg is presented in Fig. 3, which contains lithostratigraphy (Fig. 3A), clast fabric measurements (Fig. 3B), fracture orientations (Fig. 3C), grain-size distributions (Fig. 3D) and shear strength measurements with depth (Fig. 3E). The upper unit (\(C_24\) 1.8 m) contains a layer of diamicton, which overlies a laminated sequence of sand, silt and clay as well as a layer of stratified sand. Intense weathering and postglacial slumping influence the physical properties of this unit, and desiccation has made the diamicton hard and crumbly (see photograph in Fig. 4A). The clast fabric of the diamicton is very weak (Fig. 3B) and the fracture pattern (Fig. 3C) originates from wetting and drying. We interpret the whole unit to be a complex of proglacial deposits formed during decay of the Baltic ice advance, c. 14 ka BP.

A disconformity marks the transition into lodgement till with a strong SSW–NNE fabric (Fig. 3B). The upper 1.5 m of the till is very hard; it exhibits a distinct set of conjugated fractures and the uppermost 0.5 m is fissured strongly in a horizontal direction (Fig. 3C). The fractures have served as pathways of enhanced postglacial weathering, which has given this upper layer a reddish brown colour (see photograph in Fig. 4B). The hard till is underlain by soft till that also has a strong SSW–NNE clast fabric orientation (Fig. 3B). The lower till (~6 m thick) does not contain the conjugated fracture pattern of the overlying hard till, but fractures indicate a SW–NE deformational direction (Fig. 3C). The soft till is only slightly weathered and the colour is dark grey (see photograph in Fig. 4C). The hard till and the soft till have similar clast fabrics and grain-size distributions (Fig. 3B, D). Their physical difference lies predominantly in preconsolidation level. Figure 3E shows shear strength measurements in the till made.

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**Fig. 4.** Field exposure at Flakkebjerg: A. Diamicton overlying fluvial sediments, a sequence interpreted as a proglacial recessive complex (photograph is taken c. 1 m below surface). B. Weathered and strongly consolidated till layer underlying the proglacial complex (photograph is taken c. 2.5 m below surface). C. Unweathered and poorly consolidated till located beneath the overconsolidated till layer (photograph is taken c. 4 m below surface).
with a torvane. The shear strength variation with depth has a bulge-shaped profile similar to the observations from Storebælt (Fig. 2).

The physical properties of the till have been investigated through extensive laboratory tests (N. Foged, pers. comm. 2001). For instance, Clausen (1998) conducted a series of triaxial tests, uniaxial compression tests and oedometer tests on the two till layers. The upper till is strong and well consolidated; its undrained shear strength is approximately 200 kPa. The lower soft till is much weaker and has undrained shear strength of approximately 80 kPa.

The Flakkebjerg site is located in close proximity to Storebælt (see locations ‘F’ and ‘S’ in Fig. 1B) and also within the marginal moraine into which Sprogø Till is integrated. The uppermost unit found at Flakkebjerg, interpreted by us as proglacial sediments, correlates well with a thin drape (<2 m) of lateglacial and postglacial sediments that overlies the Sprogø Till in Storebælt (Foged et al. 1995). Lateglacial glaciolacustrine clays and ice-rafted dianmicton frequently overlie till from the Baltic Ice Advance (Houmark-Nielsen 1999). The geotechnical investigation by Clausen (1998) has shown that the physical properties of the Flakkebjerg till resemble the properties of the Sprogø Till in Storebælt. It is thus likely that the Baltic Ice Stream shaped the till properties at both locations.

We acknowledge that the clast fabric pattern found in the Flakkebjerg till (S–N to SSW–NNE) deviates from the generalized trend of the last Pleistocene ice advance (SE–NW, according to Houmark-Nielsen (1987)). However, this may simply reflect the local ice-flow direction, which may well have turned SW–NE towards the end of the Baltic Ice Advance. Reconstruction of ice marginal positions in SE Denmark is shown in Fig. 5, which illustrates a highly lobate terminus of the Baltic Ice Stream during its decay. It is evident from Fig. 5 that the Flakkebjerg till was exposed to SW–NE trending ice movement during deglaciation. The clast fabric orientations (Fig. 3B) and structural deformation (Fig. 3C) correspond well with this trend. The effects of marginal curvature on ice flow lines and glacial stress directions are well known. For instance, Boulton et al. (1985, 2001) showed that glacial signatures formed at terminal positions tend to overprint earlier features.

The lower part of the Flakkebjerg till has been previously interpreted as a deposit from the main glacial advance, c. 20 ka BP (Klint & Gravesen 1999). We do not share this interpretation, although there is agreement on the origin of the hard upper till. The properties of the weak lower till compare with sub-ice-stream till properties observed beneath the West Antarctic ice sheet (Kamb 2001). The properties of the strong upper till compare with predictions for stopped ice streams (Christoffersen & Tulaczyk in press (b)). It is possible that the lower till contains clasts from a source area associated with the main NE ice advance. If so, this fact may simply indicate that older till has been incorporated into the deformable till layer, which would have been 5–10 m in thickness, providing lubrication for the Baltic ice stream.

**Basal freeze-on and till consolidation**

Basal freeze-on is the process whereby subglacial water and debris are accreted onto the ice base (Alley et al. 1997). The resulting zone of accretion is typically referred to as the basal layer (Knight 1997; Lawson & Kulla 1977). A switch from basal melting to freezing takes place if the basal temperature gradient becomes sufficiently steep, either due to climatic cooling or ice thinning, or if there is a reduction in frictional heat generation at the base (Alley et al. 1997; Tulaczyk et al. 2000b). The result is a negative thermal energy balance, which is satisfied by latent heat released by freezing. Subglacial water that solidifies into a basal ice layer may be stored in a basal water system or in the pore

![Fig. 5. Geomorphologic map of the Storebælt region showing strongly curved ice marginal positions during decay of the Baltic Ice Stream, c. 14 ka BP (modified from Larsen et al. 1982). Arrows illustrate palaeo-ice-flow direction.](image-url)
spaces of the underlying till layer. If the till is coarse-grained, pore water may freeze within the till as its temperature drops below the pressure-melting point. However, the pore spaces of fine-grained till may be too small for ice crystal growth (Everett 1961; Hohmann 1997; O’Neill 1983). In this case, pore water becomes supercooled. Instead of freezing in situ, pore water is extracted out of the till. We conjecture that this subglacial mechanism is physically similar to the frost heave phenomenon, which has been extensively studied by permafrost engineers during the last several decades (Everett 1961; Miyata 1998; O’Neill & Miller 1985; Padilla & Villeneuve 1992). The extraction of pore water by freezing is a thermo-osmotic process which is also known in the permafrost literature under the term ‘cryostatic suction’ (Fowler & Krantz 1994). We treat basal freeze-on as a thermodynamically controlled process in which heat and water flows are coupled. Cryostatic suction drives water flows that may have significant implications for development of till properties.

**Thermodynamics of freezing**

Ice–water phase changes are controlled primarily by temperature and confining pressure (Hooke 1998: p. 5). The pressure-melting point is therefore commonly used as a synonym for the phase-change temperature. However, freezing of water can be influenced by additional factors. Depression of the freezing point (below the pressure-melting point) may result from ice–water surface tension, which can be significant on ice–water interfaces with micron-scale curvature (Everett 1961; O’Neill & Miller 1985). In the glaciological literature, this effect has been formerly associated with the thermodynamic equilibrium of ice and water in fine, intercrystalline veins (Raymond & Harrison 1975), but it should be similarly important for ice crystal formation within fine-grained sub-ice-stream tills (Tulaczyk 1999).

Two thermodynamic equations provide the fundamental basis for our treatment of the response of subglacial tills to freezing. The first equation specifies the surface tension, \( \Delta \rho \), which is associated with the curvature of the ice–water interface, \( dA/dV \); where \( A \) is surface area and \( V \) is volume (Tulaczyk 1999; Everett 1961; Konrad & Duquennoi 1993):

\[
\Delta \rho = p_i - p_w = \sigma_w \frac{dA}{dV}
\]

(1)

where \( p_i \) is the ice pressure, \( p_w \) is the water pressure and \( \sigma_w \) is the ice–water surface energy. Solidification of interstitial pore water requires that the ice–water interface complies closely with the surfaces of till particles, which will be surrounded by water film. Hence, \( dA/dV \approx SSA \), where SSA is the specific surface area of the sediment.

The second important thermodynamic relation is the Clapeyron equation, which specifies the temperature-pressure coupling during phase equilibrium. Liquid water freezes when the temperature and pressure satisfy the generalized form of the Clapeyron equation (O’Neill & Miller 1985; Miyata 1998):

\[
\frac{p_w - p_i}{p_i} = \frac{L_f}{273.15} T
\]

(2)

where \( p_w \) is the water pressure, \( p_i \) is the ice pressure, \( \rho_w \) is the density of water, \( \rho_i \) is the density of ice, \( L_f \) is the coefficient of latent heat of fusion and \( T \) is the temperature in °C. Solutes present in liquid water also influence the melting/freezing temperature. An increase in solute concentration has a similar effect to increasing confining pressure. This geochemical effect may be incorporated into the Clapeyron equation through an osmotic pressure term (Padilla & Villeneuve 1992). The effects of solutes are not included here because we have no constraints on palaeo-pore-water chemistry.

**Subglacial hydrology**

When ice growth is inhibited by surface tension, reduced basal temperatures are accompanied by a decrease in water pressure at the ice–water interface. This localized drop in water pressure induces non-hydrostatic hydraulic gradients that drive water flow toward the freezing interface. The hydraulic gradient can be expressed as the spatial gradient of excess water pressure, which is defined as (Domenico & Schwartz 1990: equation 4.50):

\[
u = p_w - p_h
\]

(3)

where \( p_w \) is the total pore-water pressure and \( p_h \) is the hydrostatic water pressure component. The flow of water through a porous medium is typically treated as a Darcy-type flow, with the rate of flow defined by (Domenico & Schwartz 1990: equation 4.53):

\[
q_w = -\frac{K_h \partial \nu}{\rho_w g \partial z}
\]

(4)

where \( K_h \) is the coefficient of hydraulic conductivity, \( \rho_w \) is the density of water, \( g \) is the acceleration due to gravity and \( z \) is depth coordinate. The flow rate towards the ice base can be determined by solving a one-dimensional diffusion equation (Mitchell 1993: equation 13.19):

\[
\frac{\partial u}{\partial t} = c_u \frac{\partial^2 u}{\partial z^2}
\]

(5)

where \( t \) is time, \( c_u \) is the hydraulic diffusion coefficient and \( z \) is depth coordinate.

Heat is transported by advection as well as by diffusion when pore water is flowing in a porous medium. The vertical temperature distribution, \( T \), can be obtained by solving a diffusion-advection equation
(Domenico & Schwartz 1990: equation 9.21):

\[
\frac{\partial T}{\partial t} = \kappa_{f} \cdot \frac{\partial^{2} T}{\partial z^{2}} = q_{w} \frac{\partial T}{\partial z}
\]  

where \(t\) is time, \(\kappa_{f}\) is the thermal diffusion coefficient and \(q_{w}\) is the water-flow velocity. The transport equations (4), (5) and (6), together with appropriately selected initial conditions and boundary conditions, provide the theoretical basis for treatment of the coupled flow of water and heat during basal freeze-on.

**Geotechnical properties of till**

Physical properties of till beneath a freezing ice base change in response to the pore-water extraction accompanying basal freeze-on. Till is an un lithified sediment similar to ‘soil’ in the engineering sense. For soils, the most important quantity controlling shear strength and state of consolidation is the effective stress, \(\sigma^e\), defined as (Mitchell 1993: p. 162):

\[
\sigma^e = \sigma - p_{w}
\]

where \(\sigma\) is the total gravitational stress and \(p_{w}\) is the total pore-water pressure. The shear strength of soils, \(\tau_{f}\), depends on the level of effective stress. Several strength criteria exist, but the Mohr-Coulomb failure criterion is the most commonly used (Wood 1992: p. 175):

\[
\tau_{f} = c + \sigma_{n}^e \tan \phi
\]

where \(c\) is the cohesion, \(\sigma_{n}^e\) is the effective stress and \(\phi\) is the angle of internal friction.

Soils are overconsolidated if they have been subjected to effective stresses that were greater than the in situ effective stress observed at the time of soil sampling. Strength of overconsolidated soils is higher than that of normally consolidated soils because particle compaction at the time of maximum past effective stress is partly irreversible (Mitchell 1993: pp. 212–213). In the geotechnical literature the level of overconsolidation is represented by the overconsolidation ratio (Wood 1992: p. 183):

\[
OCR = \frac{\sigma^{e}_{n,\text{max}}}{\sigma^{e}_{n}}
\]

where \(\sigma^{e}_{n,\text{max}}\) is the past maximum effective stress and \(\sigma^{e}_{n}\) is the current effective stress. The SHANSEP (Stress History and Normalized Soil Engineering Properties) method provides a correlation between stress history and present-day shear strength for natural clays. According to this method, developed at M.I.T. through empirical studies by Ladd & Foott (1974), the overconsolidation ratio may be alternatively expressed as (Wood 1992: p. 185):

\[
OCR = \left( \frac{C_{u}/\sigma^{e}_{n}}{C_{u}/\sigma^{e}_{n,\text{max}}} \right)^{1/m}
\]

where \(C_{u}\) is present-day undrained shear strength, \(\sigma^{e}_{n}\) is present-day effective stress, \(\sigma^{e}_{n,\text{max}}\) is the past maximum effective stress and \(m\) is an empirical parameter ranging between 0.7 and 0.8. Equation (10) has been verified experimentally through tests performed on a wide variety of natural clays, e.g. Mayne (1988), who compares 114 clay specimens. The method works best for sediments with low to medium sensitivity (Mitchell 1993: p. 318), such as many glacial tills.

Equations (9) and (10) can provide estimates of OCR and \(\sigma^{e}_{n,\text{max}}\) based on measurements of sediment shear strength. The SHANSEP approach offers an independent alternative to the usual techniques used to measure the degree of overconsolidation and estimate preconsolidation stress, e.g. oedometer tests (Mayne 1988). Another benefit of the SHANSEP method is that it provides a basis for relating strength variations with depth to stress history. It may be utilized to obtain estimates of past maximum effective stress from present-day effective stress and shear strength observations. By rearranging equations (9) and (10), one can express the past maximum effective stress, \(\sigma^{e}_{n,\text{max}}\), in terms of in situ shear stress, \(\sigma^{e}_{n}\), and shear strength \(C_{u}\). Data compilation by Mayne (1988) has shown that isotropic test conditions yield: \(m \approx 0.7\) and \(C_{u}/\sigma^{e}_{n,\text{max}} \approx 0.75 \sin \phi^e\), where \(\phi^e\) is the effective angle of friction. Anisotropic test conditions yield similar values of \(m \approx 0.8\) and \(C_{u}/\sigma^{e}_{n,\text{max}} \approx 0.67 \sin \phi^e\).

**Basal freeze-on induced by horizontal advection of cold ice**

In a polar ice sheet (e.g. West Antarctica), basal freeze-on can be initiated by ice thinning or sufficiently fast downward advection of cold surface ice (Alley et al. 1997; Tulaczyk et al. 2000b). However, basal freeze-on does not have to be restricted to ice streams in cold polar climates, such as the ones in West Antarctica where mean annual surface temperatures are about −25°C. The fast motion of ice streams (500 m a−1) promotes horizontal advection of cold ice from upstream to warmer regions near the ice margin (Paterson 1994: p. 220). Palaeo-ice streams that were flowing through Late Pleistocene, Northern Hemisphere ice sheets may have experienced periods of basal freezing, even in mid-latitude marginal regions with relatively mild palaeo-climates. During the Last Glacial Maximum, mean annual temperatures along the southern margin of the Fennoscandian ice sheet were approximately 12°C lower than today, i.e. near 0°C (Budd et al. 1998). Under these relatively mild, mid-latitude conditions, a switch from basal melting to basal freezing may occur only when the horizontal advection of cold ice is large enough to produce a steep basal temperature gradient. The main differences between conditions favouring
basal freeze-on beneath polar ice streams and mid-latitude palaeo-ice streams are illustrated in Fig. 6.

At the surface of a steady-state polar ice stream, a downward-directed vertical ice velocity equals the accumulation rate (Fig. 6A). A switch from basal melting to basal freezing (Fig. 6B) can be triggered by several mechanisms, e.g. ice-stream thinning or fast downward advection of cold surface ice. In response to freezing conditions, the ice stream starts to slow down because withdrawal of subglacial water decreases basal lubrication (Tulaczyk et al. 2000b; Bougamont et al. in press; Christoffersen & Tulaczyk in press (a)). The loss of frictional heat that accompanies the slow down enhances basal freezing even further. Since modern polar ice streams are within the accumulation zone, a period of ice thickening may result from the stoppage due to reduced ice discharge (Fig. 6C). We hypothesize that basal freeze-on beneath mid-latitude palaeo-ice streams is most likely to be triggered by fast horizontal advection of cold ice from upstream. This hypothesis is based on the location of these ice streams being largely within the marginal ablation zone of former ice sheets. Steady-state ice streaming in an ablation zone requires an upward ice velocity component that matches the ablation rate. Without significant horizontal advection, the temperature distribution throughout the ice column would be at the melting point. A steep basal temperature gradient is needed for freezing to occur at the bed and this can be achieved by horizontal advection, which cools the interior part of the ice stream due to high velocities associated with ice streaming (Fig. 6D). The result is a characteristically curved temperature profile (Fig. 6E). Basal freezing may also trigger ice-stream stoppage in this case, but in contrast to polar ice streams, a stopped mid-latitude ice stream will continue to thin because of the ongoing surface ablation (Fig. 6F).

Numerical modelling
We have developed a numerical model that simulates basal freeze-on and associated changes in till properties. It is a thermodynamically based one-dimensional

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Fig. 6. Schematic diagrams illustrating vertical distribution of ice temperature (solid curve) and basal conditions beneath polar ice streams (A through C) and mid-latitude ice streams (D through F). Diagram (A) shows a steady-state polar ice stream with basal melting and a temperature profile similar to observations from West Antarctica (Kamb 2001). In diagram (B), basal freezing is induced by thinning of the polar ice stream. Stoppage of a polar ice stream results ultimately in ice thickening and accretion of a frozen-on basal ice layer (C). In a steady state, a mid-latitude ice stream (D) is similar to its polar equivalent, except for the vertical ice velocity being directed upward due to surface ablation. The vertical temperature profile is dominated by horizontal advection of cold ice, which may induce freezing (E) and trigger ice-stream stoppage. A stopped mid-latitude ice stream wastes rapidly due to continuing ablation and no ice influx from upstream (F). Arrows shown in the till layer beneath the ice streams indicate water-flow direction. $\theta_{\text{c}}$ is the critical basal temperature gradient at which runaway freezing and ice stoppage start (Tulaczyk et al. 2000b). $U_x$ and $U_z$ are vertical and horizontal velocity vectors, respectively, and $a_b$ is ablation rate ($>0$) or accumulation rate ($<0$).
(vertical) model in which flows of water and heat are coupled via the Clapeyron equation (equation (2)). The model consists of two modules: a till column with a thickness of 10 m and an ice column with an initial thickness of 500 m. The till module provides a basal shear strength value to the ice-stream module. The two fundamental variables of the model are excess pore-water pressure \( u \) and temperature \( T \).

We have previously described a more complete model of basal freeze-on, which was used to simulate freeze-induced changes in the basal zone of West Antarctic ice streams (Christoffersen & Tulaczyk in press (a)). That model included solute transport and ice lens development within subglacial sediments. Here, we focus entirely on unfrozen subglacial sediments beneath a freezing ice base, not on the frozen-on basal layer itself. Our intention is to simulate the evolution of effective stress that should accompany basal freezing. We test the sensitivity of our model to poorly constrained parameters, such as the ablation rate.

**Till module**

The till module requires a high spatial resolution because the Clapeyron equation continuously readjusts the pore-water pressure beneath the freezing ice base to take into account the increasing degree of supercooling. 501 nodes represent a 10 m till column and the spatial resolution is thus 0.02 m.

The Flakkebjerg till is a sandy clay till with a specific surface area of approximately \( 5 \times 10^3 \) m\(^{-1} \) (estimated using Tulaczyk 1999: equation 12). From equation (1), the associated surface tension becomes 20 kPa. These estimates of SSA and \( \Delta p \) are just lower bounds, since they are based on the assumption that all particles are spheres (Tulaczyk 1999). In addition, measurements of grain-size distribution usually do not extend to very small grain sizes (\( \sim 0.1 \) mm). Even a small fraction of very fine clay may increase SSA and \( \Delta p \) considerably. The local ice pressure must overcome the ice overburden pressure plus the surface tension effect if ice is to form within till pore spaces (Hopke 1980; O’Neill & Miller 1985; Fowler & Krantz 1994). Since we are not interested here in the phenomenon of segregation ice growth in the till (i.e. ice lensing), we assume that the till remains unfrozen throughout our simulations. In this case basal freezing must be satisfied through withdrawal of water from the underlying till. Our previous work suggests that this simplifying assumption does not cause significant changes in the effective stress within the till. The assumption is physically admissible, since it has been observed that the stagnant Ice Stream C in West Antarctica has a basal temperature of \(-0.35^\circ\)C below the pressure-melting point (Kamb 2001), while the underlying till is unfrozen (Bentley et al. 1998). A partly frozen bed beneath the Siple ice stream, which stopped c. 500 years BP, indicates that complete freeze-up occurs several centuries after stoppage (Gades et al. 2000).

We use the Clapeyron equation (equation (2)) to specify the excess pore pressure associated with ice-water phase change at the basal temperature. The lower boundary conditions are simple balance equations related to heat flow and water flow. A geothermal heat flux enters the till column while no water flow is allowed across the lower boundary. The latter assumption corresponds to an impermeable subtill material and we choose this option because we want to isolate the system from the unknown hydraulic influences. We set the flow of water out of the till to be equal to the freezing rate, \( f \), controlled by the overall heat budget:

\[
f = \frac{K_i \theta_b - K_t \theta_t - \tau_b U_b}{L_f \rho_w}
\]

where, \( K_i \) and \( K_t \) are thermal conductivities for ice and till, \( \theta_b \) and \( \theta_t \) are temperature gradients for basal ice and till, \( \tau_b \) is the basal shear stress, \( U_b \) is the basal velocity, \( L_f \) is the coefficient for latent heat of fusion, and \( \rho_w \) is density of water.

After solving for effective stress distribution within the till, we use the SHANSEP method to predict the postglacial strength profiles of the till. From equations (9) and (10), and with \( \phi' = 34^\circ \), \( m = 0.8 \), and \( C_u \sigma_{n,\text{max}} = 0.4 \) obtained from triaxial tests by Clausen (1998), we get:

\[
C_u = 0.4 \sigma_{n,\text{max}} \left( \frac{\sigma_{n,\text{max}}}{\sigma_{n}} \right)^{0.8}
\]

where \( C_u \) is the undrained shear strength, \( \sigma_{n,\text{max}} \) is the past maximum effective stress, and \( \sigma_{n} \) is the postglacial effective stress. We obtain predictions of till strength variations with depth when \( \sigma_{n,\text{max}} \), which is calculated by the model as a function of depth, is inserted into equation (13). This allows us to make direct comparison with observed till strength profiles (e.g. Fig. 2 and Fig. 3E).

**Ice module**

The ice-stream module is needed to calculate the shear heating and the basal temperature gradient used in calculations of basal freezing rate. 51 nodes represent the vertical dimension of the ice stream, which has an initial thickness of 500 m. The gravitational stress that drives ice-stream flow is given by \( \tau_d = \rho_i g h \sin \alpha \), where \( \rho_i \) is density of ice, \( g \) is the acceleration due to gravity, \( h \) is the ice thickness and \( \alpha \) is the surface slope. We include horizontal advection of heat in the temperature model by adding a velocity-dependent cooling factor to the vertical diffusion-advection equation (Hooke 1998: equation 6.36):

\[
\frac{\partial T}{\partial t} = \frac{\kappa}{h^2} \frac{\partial^2 T}{\partial h^2} - U_z \frac{\partial T}{\partial h} + U_c \alpha \lambda
\]
Table 1. Constants describing till properties and ice-stream configuration.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value/unit</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>$2 \times 10^9$ Pa</td>
<td>Cohesion of till</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$5 \times 10^{-3}$ m$^3$ s$^{-1}$</td>
<td>Hydraulic diffusivity in till</td>
</tr>
<tr>
<td>$K_d$</td>
<td>$2.5 \times 10^{-10}$ m$^{-1}$ s$^{-1}$</td>
<td>Hydraulic conductivity in till</td>
</tr>
<tr>
<td>W</td>
<td>$18 \times 10^3$ m</td>
<td>Ice-stream half-width</td>
</tr>
<tr>
<td>$h_0$</td>
<td>$1.5 \times 10^{-3}$</td>
<td>Surface slope of ice stream</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>$7.6 \times 10^{-7}$ m$^2$ s$^{-1}$</td>
<td>Thermal diffusivity in till</td>
</tr>
<tr>
<td>$\phi$</td>
<td>34°</td>
<td>Angle of friction in till</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.01 K m$^{-1}$</td>
<td>Lapse rate</td>
</tr>
<tr>
<td>$\tau_d$</td>
<td>$6.7 \times 10^3$ Pa</td>
<td>Driving stress of ice stream</td>
</tr>
</tbody>
</table>

Here, $T$ is temperature, $t$ is time, $\kappa_i$ is the thermal diffusivity of ice, $h$ is the height above the bed, $U_x$ is vertical ice velocity, $U_y$ is horizontal velocity, $x$ is the surface slope and $\lambda$ is the lapse rate, i.e. rate of temperature change with elevation. The upper boundary condition is set to 0°C (ablation zone) and the lower boundary condition is specified by the freezing-point temperature calculated by the till module for the ice–till interface. Ice-stream velocity $U_x$ is calculated analytically from (Raymond 1996; Tulaczyk et al. 2000b):

$$U_x = \left[ \frac{1}{(1 - \tau_b/\tau_d)^{m}} \left( \frac{W}{h} \right)^{m+1} + \frac{\tau_b}{\tau_d} \right] U_d$$ (15)

Here, $W$ is the ice-stream halfwidth, $h$ is ice thickness, $\tau_b$ is basal shear strength, $\tau_d$ is the driving stress and $U_d = 2^{(1-m)} \tau_d h^{(m+1)} B_m^{m}$ is the surface velocity for ice that moves purely by internal deformation ($m$ and $B_m$ are constants of ice-flow law, modified from Raymond (1996: equation 39)).

Conservation of mass specifies the relation between ice-thickness changes and the ice fluxes that enter and leave the system. When the $x$-axis follows a flowline with ice deforming in plane strain, mass continuity requires that (Paterson 1994: p. 256):

$$\frac{\partial h}{\partial t} = -a_b - h \frac{\partial U_x}{\partial x} + U_x \frac{\partial h}{\partial x}$$ (16)

where $h$ is ice thickness, $t$ is time, $a_b$ is ablation rate ($>0$), and $U_x$ is the ice velocity. To be able to use a column ice model, rather than a full flowline model, we make two simplifying approximations: (1) $\partial h/\partial x \approx \alpha$, where $x$ is surface slope and (2) $\partial U_x/\partial x \approx K \cdot U_x$, where $K$ is a constant. We chose the value of $K$ by requiring that ice-stream thickness is in steady state when the ice stream is flowing with its initial fast velocity, i.e. $\partial h/\partial t \approx 0$. The consequence of this assumption is a vertical ice velocity, which makes up for the ablation rate during steady-state ice streaming. However, when the ice stream slows down, ice thickness starts to decrease. When the ice stream shuts down completely, ice thins at the ablation rate, $\partial h/\partial t \approx -a_b$.

Values of till parameters are derived partly from laboratory tests and partly from in situ field measurements. In making our choices, we combine observational results from Storebælt and Flakkebjerg as well as from the recent studies of West Antarctic ice streams. Values of constants describing till properties and ice-stream configuration are given in Table 1. Time-dependent parameters are listed with their initial values in Table 2. The glaciological parameters are, of course, practically unknown. Their values have been chosen in order to obtain a steady-state ice-stream solution that resembles present-day ice streams in West Antarctica while also fitting within the admittedly loose geographical constraints of the study region. The value of geothermal heat flux of the old Baltic crust is set to 0.04 W m$^{-2}$ from Artemieva & Mooney (2001). The parameters of the steady-state ice-stream condition are included in Table 2.

Results

The most important output from our numerical simulations concerns changes in till properties such as shear strength and porosity. In addition, the model tracks changes in ice and till temperature. We have investigated the evolution of till properties under different assumptions of the surface ablation rate. In the first case, we assumed a constant ablation rate of 1 m a$^{-1}$ over the whole model run lasting for 200 years (numerical experiment 1). In the subsequent two model runs we changed the ablation rate by increasing it from 1 m a$^{-1}$ to 5 m a$^{-1}$ over a 100-year period (numerical experiment 2), and then more dramatically from 1 m a$^{-1}$ to 20 m a$^{-1}$ over a 50-year period (numerical experiment 3). We are interested in exploring these different ablation scenarios because the magnitude and history of ablation during the activity of the Baltic Ice Stream is one of the most unknown model parameters. Hence, we need to investigate how sensitive the output of our model is to different ablation histories.

In all of our simulations the horizontal advection of
cold ice from upstream is sufficiently strong to ultimately cause basal freezing. The initial steady-state ice velocity is 538 m a\(^{-1}\). The subglacial till strength (equation (8)) increases significantly in response to dewatering by basal freeze-on, as seen in Fig. 7. Maximum till strength develops in the lowermost till when the ablation rate is small (experiment 1), as can be seen in Fig. 7A. As the ablation rate increases (experiment 2), the vertical strength gradient in the till reverses. This reversal stands out particularly in Fig. 7B and it is associated with an increase in cryostatic suction. Finally, maximum till strength develops near the ice–till interface when the ice stream thins rapidly (experiment 3), and this condition of ‘inverse’ strength profile can be seen clearly in Fig. 7C.

The strong influence of surface ablation rates on development of subglacial effective stresses may seem surprising. It can be better understood by looking at changes of basal temperature, pressure-melting point and the degree of supercooling at the freezing ice base. As we have already discussed, basal supercooling causes depression of pore-water pressure in the till beneath the freezing ice base. As ice thins due to surface ablation, the pressure-melting point increases. This increase of the phase transition temperature increases the supercooling at the ice base (even in the absence of additional external cooling). In Fig. 8A it is shown how a low surface ablation rate (experiment 1) is associated with basal temperatures down to \(-0.25^\circ C\) below the pressure-melting point. On the other hand, a high surface ablation rate (experiment 3) produces basal temperatures down to \(-0.36^\circ C\) below the pressure-melting point, as can be seen in Fig. 8B. In both cases, supercooling increases with time, even though the bed begins to warm. The basal warning is caused by latent heat released by the freezing process. The influence of surface ablation rates on supercooling is clearly significant and it helps explain how the rate of ice thinning may enhance development of elevated effective stresses in the till.

Figure 9 shows model results leading to the prediction of postglacial till strength distribution, as calculated from the SHANSEP method (equation (10)). Results from the numerical experiment 1 (200 model years) are presented in 10-year intervals; the numerical experiment 2 (100 years) in 5-year intervals, and the numerical experiment 3 (50 years) in 3-year intervals. Figure 9A–C illustrates changes in ice-temperature distribution for all three numerical experiments. Increases in effective stress accompanying freezing
Basal freeze-on during late Pleistocene deglaciation

The model results, which were presented in the previous section, show that mid-latitude palaeo-ice streams may have been able to transport cold ice from regions near the ice divide towards relatively warm ice-sheet margins. Horizontal advection, associated with fast ice-stream velocity (~500 m a⁻¹), produces a strongly curved temperature profile (see Fig. 6) in which surface temperature and basal temperature may be ~0°C, while the interior ice is relatively cold (up to c. −8°C in our simulations). This temperature distribution can produce steep basal temperature gradients capable of inducing basal freezing. It is thus possible that the Baltic ice stream experienced stagnation due to freezing similar to the stagnation of Ice Stream C 150 years ago in West Antarctica (Kamb 2001).

Ablation rates associated with the late Pleistocene deglaciation are unknown. If we have correctly identified the ablation rate as the primary control on development of till-strength profiles, we can speculate that our model results favour quite high ablation rates for the marginal zone of the Baltic Ice Stream. As shown in Fig. 9F, high ablation rates (>10 m a⁻¹) are associated with uneven distribution of effective stress with depth and with the highest till strength values achieved in the uppermost till. Low ablation rates (<10 m a⁻¹) are not associated with this signature because the effective stress distributes more evenly with depth (Fig. 9D). The characteristic bulge-shaped shear strength profile (Fig. 2) encountered in the uppermost till from Storebælt compares well with the model predictions generated under the assumption of fast ablation rates (>10 m a⁻¹). Fast ablation rates would favour very rapid wastage (<100 years) of the Baltic Ice Stream after cessation of its fast flow. Short-lived (several centuries) advances of ice streams and ice lobes at the southern edges of the Laurentide and the Fennoscandian ice sheets are consistent with the relatively high frequency of ice-margin fluctuations suggested by geologic evidence (Houmark-Nielsen 1987; Punkari 1995; Punkari 1997; Boulton et al. 2001) compared the dynamic behaviour of the Baltic Ice Stream during its decay to a ‘loose fire hose’.

In addition to the Danish examples discussed here, bulge-shaped till strength profiles and weak/strong till interlayering have been observed off the coast of Norway (Sættem et al. 1996). A generalized geotechnical profile from the outer mid-Norwegian continental shelf is shown in Fig. 10. The till strength variability reported in these examples is similar in character to the previously addressed Danish observations (Figs 2, 3E).
Sættem et al. (1996) have even proposed a qualitative model in which the till-strength variability observed in their data was explained by extraction of pore water during basal freezing. The Norwegian data set is also important because it shows that interlayering of strong and weak tills has developed in localities where the till sequence is well below the sea level (\( \sim 300 \text{ m b.s.l.} \)). As recognized by Sættem et al. (1996), till found at these depths is unlikely to have been exposed to permafrost processes or subaerial drying, which have been suggested as processes responsible for generation of over-consolidated till crusts (Mickelson et al. 1978). Moreover, the location of the Norwegian observation correlates with an inferred position of another palaeo-ice stream (Landvik et al. 1998; Sejrup et al. 2000).

A third relevant observation was made at the ODP site 910 located on the Yermak plateau NW of Svalbard (Shipboard Scientific Party 1995). Here, a very strong upper till layer also caps a weaker underlying till (Rack et al. 1996). These properties have been interpreted to result from consolidation beneath a large grounded ice sheet prior to 660 ka BP (Flower 1997; Sejrup et al. 2000). The geotechnical similarity to our data results suggests that the till at the ODP site 910 may have also

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**Fig. 9.** Modelled changes in ice and till properties during ice-stream stagnation. (Left) Ablation rate of 1 m a\(^{-1}\) over a 200-year period. (Middle) Ablation rate increasing from 1 m a\(^{-1}\) to 5 m a\(^{-1}\) over a 100-year period. (Right) Ablation rate increasing from 1 m a\(^{-1}\) to 20 m a\(^{-1}\) over a 50-year period. A–C. Changes in ice-temperature distribution during glacial decay. D–F. Increase in subglacial effective stress in the till. G–I. Evolution of postglacial till strength profiles predicted with the SHANSEP method using the effective stress changes shown above as input. Arrows indicate trend of change with time.
been exposed to sub-ice sheet (but not necessarily sub-ice stream) freezing.

Although we are encouraged by the results of our model, we acknowledge that several assumptions have been made in order to model palaeo-ice-stream dynamics. The geometry of the Baltic Ice Stream is poorly constrained and this is problematic because ice thickness, ice-stream width and ice-surface slope are key parameters in quantifying the dynamics of this palaeo-ice stream. This uncertainty may have limited influence on our efforts to model till overconsolidation, since the relevant till property changes take place when the ice stream is already in a stagnant mode. This leaves us with one main palaeo-glaciological assumption: that the horizontal advection of cold ice from upstream was fast enough (~500 m a\(^{-1}\)) to produce high basal temperature gradients and induce basal freezing. Although there are no direct constraints that could tell us what the velocity of the Baltic Ice Stream was, this basic assumption stems from the observation that modern ice streams and palaeo-ice streams are frequently associated with such high velocities (Anderson et al. 2002; Punkari 1995; Shipp et al. 1999; Stokes & Clark 2001).

Conclusions

We have developed a numerical model that couples ice-stream dynamics to transient changes of sub-ice-stream till properties in a relatively simple, one-dimensional approach. In doing so, we utilize concepts from frost heave simulations adapted from permafrost engineering. The model includes pore-water flow driven by hydraulic gradients that are induced by supercooling of the ice base. This freezing-point depression arises when ice-water surface tension inhibits growth of ice in fine-grained subglacial tills. The supercooling-driven depression of pore-water pressure causes an increase in subglacial effective stress as well as an increase in till strength. Using the SANSHEP method, we convert simulated subglacial effective stress histories into postglacial till strength profiles.

Our model shows that horizontal advection of ice can trigger a switch from basal melting to basal freezing, even under relatively warm, mid-latitude ice-sheet conditions characterized by air temperatures around 0°C. Effective stresses induced by basal freeze-on distribute evenly throughout the simulated till domain if the surface ablation rate is low (<10 m a\(^{-1}\)) and the till consolidates relatively uniformly with depth. However, if the surface ablation rate is high (>10 m a\(^{-1}\)), the upper part of the till experiences higher effective stresses than the lower part of the till, and in this case the till consolidates unevenly with depth. The result is a geological sequence where strong and well-consolidated till overlies weak and poorly consolidated till. A bulge-shaped shear strength variation with depth characterizes this geological sequence. Our model is able to reproduce bulge-shaped till strength profiles observed at two separate locations overridden by the Baltic Ice Stream. Such bulge-shaped till strength profiles may be a signature of palaeo-ice-stream stagnation triggered by basal freezing and followed by rapid retreat/wastage (50–100 years) due to high surface ablation rates (>10 m a\(^{-1}\)).

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References


Bentley, C. R., Lord, N. & Liu, C. 1998: Radar reflections reveal a


