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Lecture 1

Contract Enforcement

The objective of this course is to analyse the interaction between the lender(s) and wealth-less borrower(s) in the context of credit markets. In this lecture, we examine the relation between borrower’s limited ability to enforce contracts and strategic or involuntary default by the borrower.

1.1 The Setup

In a typical credit market scenario, a lender offers the borrower a contract which specifies the following:

1. The amount he is ready to loan\(^1\)
2. The duration of the loan
3. The repayment obligation or the interest rate charged on the loaned amount.

Once the loan duration is over, the borrower could either meet the repayment obligation or default on the loan. If she chooses to default, it could be due to the following two reasons.

**Involuntary Default:** The project fails and produces insufficient output to meet the repayment obligations.

**Strategic Default:** The project produces sufficient output to meet the repayment obligations but the borrower *chooses* not to repay.

Even though credit markets are notorious for information problems, in the case of involuntary or strategic default, there is no information problem. The

\(^1\)Let's assume for a particular project
lender may know that the borrower is choosing not repay and may not be able to enforce repayment due to limited ability to enforce contracts.2

Unless stated otherwise, we assume throughout the lectures that the lender and the borrower(s) are both risk-neutral.

Project: A borrower’s project is always such that it requires an investment of 1 unit of capital at the start of period 1 and produces stochastic3 output $x$ at the end of period 1. The borrower has zero wealth and can thus only initiate the project if the lender agrees to lend to her.

Contract: The lender offers the borrower(s) a contract whereby each borrower receives 1 unit of capital investment for the project. The contract specifies the borrower’s total repayment obligation $r (> 1)$ once the project output is realised. To make the model extremely stark, we assume that the borrower can always meet the repayment obligations.

We assume that repayment is an all or nothing decision, i.e., the borrower either repays $r$ or declares default, in which case she pays nothing. Thus, once the project has been completed and the project output has been realised, the borrowers arrive upon their decision regarding the repayment of the loan by comparing the consequence of repayment with the consequence of default.

Enforcement: In an ideal world, the lender would have an unlimited ability to enforce contacts (read punish the borrower for defaulting) and would obtain repayment with certainty.4 With limited enforcement capability, the lender would only be able to obtain repayment in the cases where the punishment meted out by the lender exceeds the borrower’s benefit from defaulting.

Below we first set out the individual lending case and then explore ways in which the lender can harness the borrower’s ability to social sanction each other by lending to groups of borrowers. The lender’s objective remains to maximise the repayment rate by using local social sanctions amongst the borrowers to leverage his own limited ability to punish them.

1.2 Strategic Default:

This section presents a simplified version of the Besley and Coate (1995) model.

Project: 1 unit of capital investment yields $x$. $x$ is distributed on $[\underline{x}, \bar{x}]$ ac-

---

2In the lectures that follow, we would look at the information problem between the lender and borrower. The imperfect information models would be analysed under the principal agent framework where we would use the term principal and lender and agent and borrower interchangeably.

3the output is a random variable with its support and distribution function specifically defined for each model we address.

4Recall, we have assumed away involuntary default by assuming that the borrower can always repay.
Strategic Default: Contract Enforcement

according to the distribution function $F[x]^\text{5}$.

**Definition 1.** Penalty Function $p(x)$: the output contingent penalty that the lender can impose on the borrower(s) once the project has been completed and the output $x$ has been realised. We assume that $p'(x) > 0$, $p''(x) \leq 0$ and $p(x) < x \forall x$.

**Definition 2.** Threshold Function $\phi(r)$: Given $r$, it gives the threshold output beyond which the borrower would choose to repay. Conversely, if the project output is below this threshold output, the borrower would choose to default strategically. It follows that $\phi'(r) > 0$, $\phi''(x) \geq 0$ and $\phi(r) > r \forall r$.

- Under individual lending, the loan repayment has the following pattern

\[ F(x) = 0 \text{ and continuous on } [a, b]. \]
Strategic Default: Contract Enforcement

Figure 1.2: Default and Repayment Regions

<table>
<thead>
<tr>
<th>Case</th>
<th>Project output range</th>
<th>Loan status</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Greater than $\phi(r)$</td>
<td>Repay</td>
</tr>
<tr>
<td>B</td>
<td>Otherwise</td>
<td>Default</td>
</tr>
</tbody>
</table>

Thus, given $r$, we know that the borrower defaults in the range $(x, \phi(r))$ and repays in the range $(\phi(r), \bar{x})$. As $r$ increases, the default range increases and the repay range decreases.

Individual Lending Repayment Rate:

$$\Pi_I(r) = 1 - F[\phi(r)]$$

$$\Pi'_I(r) < 0$$

1.2.1 Group Lending without Social Sanctions

Groups are composed of two ex ante identical, borrowers 1 and 2. (B1 and B2 henceforth)

Group Contract: The group gets 2 units of investment capital for the project and has a collective repayment obligation of $2r$ once the projects are completed. Both borrowers are penalised if this repayment obligation is not met.

You would notice that the borrowers in the group are jointly liable for the repayment, i.e., they are collectively responsible for repaying $2r$. Thus, the borrower’s penalty is contingent not just on his own output realisation but also on the output realisation of her peer.

Timeline:

- Project returns $x_1$ and $x_2$ are realised.\(^6\)

Stage 1 Borrowers decide simultaneously whether to repay $r$ or not.

Stage 2 If the decision is unanimous, payoffs are as follows:

- Both choose to repay: $x_1 - r$, $x_2 - r$
- Both choose not to repay: $x_1 - p(x_1)$, $x_2 - p(x_2)$

- When the decision is not unanimous, the borrower who decided to repay in the first stage can revise her decision by either paying $2r$ or 0.

E.g., if B1 chooses repay and B2 chooses not repay in stage 1, then B1’s final payoffs are:

\(^6\)Output is common knowledge amongst the borrowers but unknown to the lender
Strategic Default:

Contract Enforcement

Stick to the decision and repay: \( x_1 - 2r, \ x_2 \)

Revise decision and default: \( x_1 - p(x_1), \ x_2 - p(x_2) \)

− Under group lending, the loan repayment has the following pattern:\(^7\):

<table>
<thead>
<tr>
<th>Case</th>
<th>Project output range</th>
<th>Group Loan status</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>At least one greater than ( \phi(2r) )</td>
<td>Repaid</td>
</tr>
<tr>
<td>D</td>
<td>Both between ( \phi(r) ) and ( \phi(2r) )</td>
<td>Repaid</td>
</tr>
<tr>
<td>E</td>
<td>Otherwise</td>
<td>Not Repaid</td>
</tr>
</tbody>
</table>

Figure 1.3: Advantages and Disadvantage of Group Lending

Group Lending Repayment Rate:

\[
\Pi_G(r) = 1 - \left\{ F[\phi(2r)] \right\}^2 + \left\{ F[\phi(2r)] - F[\phi(r)] \right\}^2
\]

Figure 1.4 allows us to compare group lending with individual lending.\(^8\)

− Under Area 1 (Area 4), B1 (B2) would have defaulted under individual lending. The loans are repaid under group lending.

\(^7\)Under Case D, non-repayment is a possibility if both borrowers believe that the other will not repay. This coordination failure can easily be assumed away by allowing the borrowers to renegotiate after stage 1.

\(^8\)Area 5: Official penalty is not strong enough to give either borrower incentive to repay. Area 6: Both borrowers prefer repaying \( r \) to incurring official penalties. Area 7: The group always repays back since repaying \( 2r \) is better than incurring official penalties.
Under Area 2 (Area 3), B2 (B1) would have repaid under individual lending but does not pay under group lending due to joint liability.

### 1.2.2 Group Lending with Social Sanction

In the previous sections, there was no cost to a borrower from defaulting other than the lender’s penalty. We now analyse how under group lending, the group member’s ability to social sanction each other can be used to amplify the effect of the lender’s penalty.

- In group lending without social sanction, the group that defaulted was in repayment Case E. We can explore Case E further as follows:

<table>
<thead>
<tr>
<th>Case</th>
<th>Project output range</th>
<th>Group Loan status</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>$x_m &lt; \phi(r); \phi(r) \leq x_n &lt; \phi(2r)$</td>
<td>Maybe Repaid</td>
</tr>
<tr>
<td>E2</td>
<td>Both less than $\phi(r)$</td>
<td>Not Repaid</td>
</tr>
</tbody>
</table>

Where $x_m$ and $x_n$ are the actual realised values of the random variables $x_1$ and $x_2$, the borrower’s respective outputs.

The group members impose a negative externality on each other in Case E1, i.e., one group member would like to pay off her own loan but defaults because her peer is going to default.

**Definition 3.** *If a group member imposes a negative externality on her peer, she faces a social sanction $s$ in response.*

![Figure 1.4: threshold Output with Social Sanctions](image)

To keep matter simple, we assume that $s$ is a constant.
Strategic Default: Contract Enforcement

\[ r \leq p(x) + s \quad \text{Repay} \]
\[ r > p(x) + s \quad \text{Default} \]
\[ \Rightarrow \quad \phi(r-s) \leq x \quad \text{Repay} \]
\[ \phi(r-s) > x \quad \text{Default} \]

- In group lending with social sanctions, the group’s repayment decision in Case E is as follows:

<table>
<thead>
<tr>
<th>Case</th>
<th>Project output range</th>
<th>Group Loan status</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1a</td>
<td>( \phi(r-s) \leq x_m &lt; \phi(r) ); ( \phi(r) \leq x_n &lt; \phi(2r) )</td>
<td>Repaid</td>
</tr>
<tr>
<td>E1b</td>
<td>( x_m &lt; \phi(r-s) ); ( \phi(r) \leq x_n &lt; \phi(2r) )</td>
<td>Not Repaid</td>
</tr>
<tr>
<td>E2</td>
<td>Both less than ( \phi(r) )</td>
<td>Not Repaid</td>
</tr>
</tbody>
</table>

Figure 1.5: Advantages and Disadvantage of Group Lending

The repayment rate under group lending with social sanctions is given by:

\[
\Pi_{G,S}(r) = 1 - \left\{ F[\phi(r)] \right\}^2 - 2 \int_{\phi(r)}^{\phi(2r)} F[\phi(r-s)]dF(x) \\
= 1 - \left\{ F[\phi(r)] \right\}^2 - 2F[\phi(r-s)] \left\{ F[\phi(2r)] - F[\phi(r)] \right\}
\]

The second term represents the likelyhood that both borrowers realise a return which is below \( \phi(r) \) and hence neither has an interest in repaying the loan. The third term represents the case where one borrower would like to repay but the other cannot be induced to repay, although she is being socially sanctioned by her peer.
Under harsh social sanctions, i.e., \( s \rightarrow r \), the repayment rate reduces to

\[
\lim_{s \rightarrow r} \Pi_{GS} = 1 - \{F[\phi(r)]\}^2
\]

It should be easy to check that \( \Pi_{GS} \) is greater than \( \Pi_G \) and \( \Pi_I \). Thus, joint liability raises repayment rate if the social sanctions are strong enough.

### 1.3 Related Ideas

One of the problems faced by borrowers is that the microfinance lenders may over-punish the borrowers, i.e., punish the borrower even when she is unlucky and defaults involuntarily. This is a deadweight loss. In an interesting paper, Rai and Sjöström (2004) analyse the implication of allowing the borrowers to cross-report on each other. They assume that even though the borrower has unlimited enforcement ability (i.e., extremely high punishment), the lender is unable to distinguish between involuntary and strategic default. Thus, if the lender is not able to verify the state, the lender would over punish under both involuntary and strategic default. Punishment for involuntary default is a deadweight loss.

Many microfinance programmes allow borrowers to cross report on each other once the output has been realised. Cross-reporting allows the lender to gather information on a problem borrower’s output \(^{10}\) by soliciting reports from her peers and showing leniency when all reports agree with each other. Rai and Sjöström (2004) show that this reduces the deadweight loss.

In a similar vein, Jain and Mansuri (2003) suggest that the microfinance lenders like to use the information and enforcement capability of the local moneylender. They do so by requiring that the borrowers repay in tightly structured installments (which begin very soon after the disbursement of the loan). This induces the borrowers to borrow from the local moneylender in order to repay the microfinance lender. Thus, the lender leverages his own capabilities using the local moneylender’s capabilities.

### 1.4 Summing Up

To summaries, if the lender wants to enforce the contracts with her limited ability to impose penalty on the delinquent borrower(s).

\(^{10}\)one that defaults
below which they choose to default on the repayment of the loan and attract the lender’s penalty. This gives rise to strategic defaults, i.e., individual borrowers default even when their output is on one hand sufficiently high to meet the loan repayment obligations but on the other hand below the above mentioned threshold.

Joint-Liability Group-Lending: Joint liability enables the lender to use the local intra-group social sanctions to extract repayment when the group’s output is greater than its repayment obligations but one of the group members has the incentive to strategically default.

Besley and Coate (1995) show that the advantage of group lending is that a group member with really high project returns can pay off the loan of a partner whose project does very badly. This is a kind of insurance for the borrowers.

The disadvantage of group lending is that a moderately successful borrower may default on her own repayment because of the burden of having to repay her partner’s loan. However, if social ties are sufficiently strong, the net effect is positive because by defaulting wilfully, a borrower incurs sanctions from both the bank and the group members. With sufficiently close social ties amongst the group members, the repayment under group lending is higher than under individual lending.

The insight of the Besley and Coate (1995) model is that in absence of strong social sanctions, there is a tradeoff between group and individual lending repayment rate. As social sanctions increase, the balance starts titling in favour of group lending.
In this lecture, we look at the problem of private information. The potential borrowers are socially connected and live in an informationally permissive environment, where they know themselves and each other very well. The lender is not part of this information network and thus does not have access to the borrowers’ information network.

The potential borrowers differ in their respective inherent characteristics or ability to execute projects. These characteristics determine the borrower’s chances of successfully completing the project. The borrowers are fully aware of their own characteristics as well as the characteristics of other borrowers around them. The lender’s problem is that the borrowers possess some private or hidden information, which is relevant to the project. The lender would like to extract this information. The only way he can do that is through the loan contracts he offers the borrowers. We set out the main ideas in the adverse selection literature and then examine how the lender can improve his ability to extract information by offering inter-linked contracts to multiple borrowers simultaneously.

The lender could offer the contract to groups instead of individuals. This would allow him to inter-link a borrower’s payoff by making it contingent on her own and her peers’ payoff. The part of the payoff that is contingent on her peer’s outcome is the joint liability component of the payoff. We show that this joint liability component is critical in dissuading the wrong kind and encouraging the right kind of borrowers.

The Principal-Agent Framework: We use the principal-agent framework to analyse the problem of lending to the poor. Usually, a principal is the uninformed party and the agent the informed party, the party possessing the private or hidden information. This information needs to have a bearing on the task.
the principal wants to delegate to the agent. The information gap between
the principal and the agent has some fundamental implication for the bilat-
eral or multi-lateral contract they may choose to sign. Further, even though
the agent(s) may renge on her contract, the assumption always is that the principal
never does so.

In the context of the credit markets, the term principal is used interchangeably
with lender and the term agent is used interchangeably with borrower. Unless
stated otherwise, we assume throughout the lectures that the lender and the
borrower(s) are both risk-neutral.

**Project**

A project requires an investment of 1 unit of capital and at the start of period
1 and produces stochastic output \( x \) at end of period 1. All borrowers have
zero wealth and can thus only initiate the project if the lender agrees to lend
to her. As is typical in a adverse selection model, the value, as well as the
stochastic property of the output depends on the type of borrower undertaking
the project. To keep matters simple, we assume that the project produces a
output with strictly positive value when it succeeds and zero when it fails.

A project undertaken by a borrower of type \( i \) produces an output valued at
\( x_i \) when it succeeds and 0 when it fails. Further, the probability of the project
succeeding is contingent on the borrower types. The project succeeds and fails
with probability \( p_i \) and \( 1 - p_i \).

**The Agents**

We have an world with two types of agents or borrowers, the *safe* and the *risky*
type. The projects that risky and safe types’ undertake succeed with probability
\( p_r \) and \( p_s \) respectively with \( p_r < p_s \). That is, the risky type succeeds less often
then the safe type. The proportion of risky type and safe type is \( \theta \) and \( 1 - \theta \)
respectively in the population. The expected payoff of an agent of type \( i \) is
given by

\[
U_i(r) = p_i(x - r).
\]

Given that interest is paid only when the agents succeed, the safe agent’s utility
is more interest sensitive as compared to the risky agent’s utility since he
succeeds more often.\(^1\) Both types are impoverished with no wealth and have a
reservation wage of \( u \).

\(^1\)This leads to the safe types utility having a steeper slope than the risky types in the
figures ahead.
The Principal

The principal’s or the lender’s opportunity cost of capital is $\rho$, i.e., he either is able to borrow funds at interest rate $\rho$ to lend on to his clients or has an opportunity to invest his own funds in a risk-less asset which yields a return of $\rho$.

We assume that the lender is operating in a competitive loan market and can thus make no more than zero profit. This implies that the lender lends to the borrowers at a *risk adjusted interest rate*. The lender’s zero profit condition $\rho = p_i r$ ensures that on a loan that has a repayment rate of $p_i$, the interest rate charged is always

$$r_i = \frac{\rho}{p_i} \quad (2.1)$$

It is important to note that competition amongst the lenders ensures that a particular lender can only choose whether or not to enter the market. He is not able to explicitly choose the interest rate he lends at. He always has to lend at the risk adjusted interest rate, at which he makes zero profits. Given that $p_r$, $p_s$, $\theta$ and $\rho$ are exogenous variables, we can take the respective risk adjusted interest rate to be exogenously given as well.

In the lecture on moral hazard we discuss the conditions under which making the assumption of zero profit condition would be justified. We find that this assumption is not critical at all. What matter is the surplus a project creates. The assumptions on loan market just determine the way in which this surplus is shared between the lender and the borrower.

Concepts

*Repayment Rate:* The repayment rate on a particular loan is the proportion of borrowers that repay back.\(^2\) If the lender is able to ensure that he lends only to the risky type, his repayment rate is $p_r$. Similarly, it is $p_s$ if he only lends to the safe type. If he lends to both type, his average repayment rate is $\bar{p} = \theta p_r + (1 - \theta)p_s$.

Pooling and Separating Equilibrium: If the lender is not able to instinctively distinguish the agent’s types, then the only way in which he can discriminate between the two types is by inducing them to self select and reveal their hidden information.

In a pooling equilibrium, both types of agents accept the same loan contract. Consequently, both types of agents are pooled together under the same loan contract.

\(^2\)Put another way, given the past experience, it is also the lender’s bayesian undated probability that the borrowers of future loans would repay.
contract. Conversely, in a separating equilibrium, a particular loan contract is accepted by only one type. The lender is able to induce the agents to reveal their private information by self selecting into different types of loan contracts.

Socially Viable Projects Socially viable projects are the ones where the output exceeds the opportunity cost of labour and capital involved in the project.

\[ p_i x \geq \rho + u \quad i = r, s; \]  
\[ (2.2) \]

That is the expected output of the project exceeds the reservation wage of the agent and the opportunity cost of capital invested in the projects. In an ideal (read first best) world, all the socially viable projects would be undertaken and that lays the perfect information benchmark for us. What is of interest to us is how the problems associated with imperfect information restrict the range of projects that remain feasible.

2.1 Individual Lending

In this section we look at individual lending and explore the implication of hidden information on the optimal debt contracts offered by the lender to the borrower.

2.1.1 First Best

In the first best world, the lender can identify the type he is lending to and can tailor the contract accordingly. Consequently, he would lend to the safe type at the interest rate \( r_s = \frac{\rho_s}{p_s} \) and to the risky type at the interest rate \( r_r = \frac{\rho_r}{p_r} \). Given that \( p_r < p_s \), i.e., the risky type succeeds and repays back less often, the risky type gets the loan at a higher interest rate as compared to the safe type. 

(Figure 2.1) The lender’s average or pooling repayment rate across his cohort of risky and safe borrowers is given by

\[ \bar{p} = \theta p_r + (1 - \theta)p_s \]  
\[ (2.3) \]

2.1.2 Second Best

In absence of the ability to discriminate between the risky type and the safe type agents, the lender has no option but to offer a single contract. This contract may either attract both types or just attract one of the two types. The lender has to makes sure that any contract that he offers satisfies the following conditions.
1. **Participation Constraint**: This condition is satisfied if the lender provides the borrower sufficient incentive to accept the loan contract.

\[ U_i(r_r) > u \]

2. **Incentive Compatibility Constraint**: In a separating equilibrium, this condition is satisfied if each borrower type has the incentive to take the contract meant for her and does not have any incentive to pretend to be the other type. Let’s say that the lender’s contract has two components, the interest rate \( r \) and some other component \( \vartheta \). The lender can now offer two contracts. He can offer a contract \( (r_r, \vartheta_r) \) meant for the risky type and a contract \( (r_s, \vartheta_s) \) for the safe type. We would get a separating equilibrium if the following conditions hold.

\[
U_r(r_r, \vartheta_r) > U_r(r_s, \vartheta_s) \\
U_s(r_s, \vartheta_s) > U_s(r_r, \vartheta_r)
\]

The first equation just says that the risky type weakly prefers taking a contract \( (r_s, \vartheta_s) \) than a contract at interest rate \( (r_r, \vartheta_r) \). Similarly, the second equation is satisfied when the safe type weakly prefers taking a contract at interest rate \( (r_s, \vartheta_s) \) over one at interest rate \( (r_r, \vartheta_r) \).

Of course, this would only work if \( \vartheta_i \) entered the borrower’s utility function. If it did not, the lender would be left with a contract that specifies the
interest rate $r$ and would offer only one interest rate to both types.\textsuperscript{3} At this interest rate, either both types would accept the contract or only one type would accept the contract.

3. **Break even condition:** Break-even condition is the lower bound on the profitability, that is, the lender’s profit should not be less than zero. Turns out the competition in the loan market puts an upper bound on profits and ensures that profits cannot be more than zero. *(Zero Profit Condition)* Thus, in this case the lender’s break even condition and zero profit condition give us a condition that binds with equality.

Turns out, the precise course of action the lender would take depends on the stochastic properties of project. Specifically, it depends on the first and second moments.

### 2.1.3 The Under-investment Problem

Stiglitz and Weiss (1981) analyse the problem under the assumption that both types’ project have the same expected output and the risky type produces an output of a higher value than the safe type since he succeeds less often.

\[ p_rx_r = p_sx_s = \hat{x} \]
\[ p_r < p_s \Rightarrow x_r > x_s \]

It also follows from the assumption that the lender can lend to the safe type in only the pooling equilibrium. Any interest rate that satisfies the safe type’s participation constraint also satisfies the risky types participation constraint. This is because the safe type’s payoff is always lower than the risky type’s payoff at any given positive interest rate.

\[ U_s(r) < U_r(r) \quad \forall \ r > 0; \]

Consequently, the safe type can only borrow in a pooling equilibrium. With the assumption in (2.4), she will never ever participate in the separating equilibrium. This implies that there are some of safe type’s projects that are not financed, even though they are socially viable, due to the problems associated with hidden information.\textsuperscript{4} The safe type would only participate in the pooling equilibrium if her participation constraint is satisfied at the pooling interest rate.

\textsuperscript{3}If the lender offered two interest rates, all rational borrowers would choose the lower one.

\textsuperscript{4}This is the range of safe type’s projects that would have been financed in the first best but do not get financed in the second best.
\[
U_s(\bar{r}) = \hat{x} - \bar{r} \geq u
\]
Substituting for the value of \(\bar{r}\) using (2.1) and (2.3), this condition becomes
\[
\hat{x} \geq \frac{p_s}{p} \rho + u. \tag{2.5}
\]
(2.5) gives us a lower bound on the expected output of the projects that get financed. Since \(p_s > \bar{p}\), we find that there are projects that would not be financed even though they are socially viable.\(^6\)
\[
\hat{x} \in \left[ \rho + u, \left( \frac{p_s}{p} \right) \rho + u \right]
\]
If (2.5) is not satisfied, the lender would lend only lend to the risky type in a separating equilibrium. Please check that all risky type’s socially viable projects get financed either in the pooling or the separating equilibrium.

Consequently, the under-investment problem in Stiglitz and Weiss (1981) is

---
5The pooling repayment rate is a weighted sum of risky and safe type’s respective repayment rates and thus would always be lower than the higher of the two repayment rates, the safe type’s repayment rate.

6Note that the projects that are not financed are on the lower end of the productivity scale. If the projects are productive enough, all socially viable projects get financed.
that there are some safe type’s project that do not get financed even though they are socially viable. In terms of their productivity, these projects on the lower end of the socially viable projects. They are below the threshold level defined by (2.5) but above the threshold given by (2.2). Conversely, all risky type’s socially viable projects get financed.

2.1.4 The Over-investment Problem

De Mezza and Webb (1987) analyse the case when the two types produce identical outputs when they succeed and fail. Consequently, the safe type’s project has a higher productivity than the risky type’s project.

\[ p_r x < p_s x \]  \hspace{1cm} (2.6)

It follows that for an interest rate in the relevant range, the safe type’s payoff is always higher than the risky type’s payoff.

\[ U_s(r) > U_r(r) \quad \forall \ r \in [0, x]; \]

The risky type would stay in the market till her participation constraint below is satisfied.

\[ U_r(\bar{r}) = p_r (x - \bar{r}) \geq u \]

Substituting for the value of \( \bar{r} \) using (2.1) and (2.3), this condition becomes

\[ p_r x \geq \frac{p_r}{\rho} \rho + u. \]  \hspace{1cm} (2.7)

Given that \( p_r < \bar{p} \), the threshold given by (2.7) is below the social viability threshold given by (2.2). This implies that the risky type are able to undertake projects that are not socially viable. Risky type’s projects with expected output in the range \( p_r x \in \left[ \left( \frac{p_r}{\bar{p}} \right) \rho + u, \ \rho + u \right] \) are financed even though they are not socially viable. The risky types in this case are able to borrow because they are being cross-subsidised by the safe type. The over-investment problem in De Mezza and Webb (1987) is that there are risky type’s projects that are financed even though they are not socially viable and have a negative impact on the social surplus. This happen because the lender is not able to discriminate between a borrower of a safe and risky type due to the hidden information they possess. The over-investment projects are the ones that do not satisfy the social viability condition defined by (2.2) and are yet above the threshold defined by (2.7) which allows them to satisfy the risky type’s participation constraint. The under and over-investment problem is summarised in Figure 2.4.
2.2 Group Lending with Joint Liability

This section is a simplified version of Ghatak (1999) and Ghatak (2000). The lender lends to borrowers in groups of two. The contract that the lender offers the group is such that the final payoffs are contingent on each other’s outcome. Consequently, the members within the group are jointly liable for each other’s outcome. If a borrower succeeds, she pays the specified interest rate \( r \). Further, if her peer fails, she is required to pay an additional joint liability component \( c \). The lender offers a joint liability contract \((r, c)\) where he specifies

- \( r \): The interest rate on the loan due if the borrower succeeds.
- \( c \): The additional joint liability payment which is incurred if the borrower succeeds but her peer fails.

Of course, if a borrower’s project fails, the limited liability constraint applies and the borrower does not have a pay anything.
A borrower’s payoff in the group lending is given by.

\[ U_{ij}(r, c) = p_i p_j (x_i - r) + p_i (1 - p_j) (x_i - r - c) \]

\[ = p_i (x_i - r) - p_i (1 - p_j) c \]

Given the group contract \((r, c)\) on offer, lender requires that the borrowers self-select into groups of two before they approach him for a loan.

**Definition 4** (Positive Assortative Matching). *Borrowers match with their own type and thus the groups are homogenous in their composition.*

**Definition 5** (Negative Assortative Matching). *Borrowers match with other type and thus the groups is heterogenous in its composition.*

With positive assortative matching, the groups would either have both safe types or both risky types. With negative assortative matching each group would have one safe type and one risky type.

**Proposition 1** (Positive Assortative Matching). *Joint Liability contracts of the type given above lead to positive assortative matching.*

To see this, let’s examine the process of matching more closely. It is evident that due to the joint liability payment \(c\), everyone want the safest partner they can get. The safer the partner, the lower the probability of incurring the joint liability payment \(c\) due to her failure. We need to examine the benefits accruing to the risky type by taking on a safe peer and the loss incurred by the safe type by taking on a risky peer.

\[ U_{rs}(r, c) - U_{rr}(r, c) = p_r (p_s - p_r) c \]  \hspace{1cm} (2.8)

\[ U_{ss}(r, c) - U_{sr}(r, c) = p_s (p_s - p_r) c \]  \hspace{1cm} (2.9)

\[ p_s (p_s - p_r) c > p_r (p_s - p_r) c \]  \hspace{1cm} (2.10)

(2.8) gives us the gain accruing to the risky type from pairing up with a safe type in stead of a risky type. (2.9) gives us the loss incurred by a safe type from pairing up with a risky type in stead of another safe type.

\[ U_{ss}(r, c) - U_{sr}(r, c) > U_{rs}(r, c) - U_{rr}(r, c) \]  \hspace{1cm} (2.11)

Turns out, the safe type’s loss exceeds the risky type’s gain. The risky type would not be able to bribe the safe type to pair up with her. Joint liability contract leads to positive assortative matching where a safe type pairs up with another safe type and the risky type pairs up with another risky type.

**Proposition 2** (Socially Optimal Matching). *Positive assortative matching maximises the aggregate expected payoffs of borrowers over all possible matches*
(2.12) is obtained by rearranging (2.11). This implies that positive assortative matching maximises the aggregate expected payoff of all borrowers over different matches.

### Indifference Curves

The indifference curve of borrower type $i$ is given by

$$ U_{ij}(r,c) = p_i(x_i - r) - p_i(1 - p_j)c = \bar{k} $$

$$ \left[ \frac{dc}{dr} \right]_{U_i = \text{constant}} = -\frac{1}{1 - p_i} $$

This is because the safe type does not mind the joint liability payment $c$ because she is paired up with a safe type. She would like to get a lower interest rate and
does not mind a higher joint liability payment in exchange. Conversely, the risky type dislikes the joint liability payment comparatively more. This is because she is stuck with a risky type borrower and incurs the joint liability payment more often than the safe type. She would prefer to have a lower joint liability payment down and does not mind the resulting increase in interest rate. The lender can use the fact that the safe groups and the risky groups trade off the joint liability payment and interest rate payment at different rates to distinguish between the two.

The Lender's Problem

Now that there are two instruments in the contract, namely $r$ and $c$, the lender can use the fact the two types trade off $r$ with $c$ at a different rate to induce them to self select into contracts meant for them. The lender offers contracts $(r_r, c_r)$ and $(r_s, c_s)$ and designs the contracts in such a way that the risky type borrowers take up the former and safe type take up the latter contract. The lender offers group contracts $(r_r, c_r)$ and $(r_s, c_s)$ that maximises the borrowers payoff subject to the following constraint:

\[
\begin{align*}
& r_r p_r + c_r (1 - p_r) p_r \geq \rho \\
& r_s p_s + c_s (1 - p_s) p_s \geq \rho
\end{align*}
\]

$L-ZPC_i$ is the lender’s zero profit condition for borrower type $i$, $PC_i$ the Participation Constraint for type $i$, $LLC_i$ the limited liability constraint for type $i$ and $ICC_{ii}$ the incentive compatibility constraint for group $i, i$.

To discuss the optimal contract that allows the lender to separate the types, we need to define the $(\hat{r}, \hat{c})$. This is at the point where $(L-ZPC_s)$ and $(L-ZPC_r)$ cross.

Separating Equilibrium in Group Lending

**Proposition 3** (Separating Equilibrium). For any joint liability contract $(r, c)$

i. if $r_s < \hat{r}$, $c_s > \hat{c}$, then $U_{ss}(r_s, c_s) > U_{rr}(r_s, c_s)$

ii. if $r_r > \hat{r}$, $c_r < \hat{c}$, then $U_{rr}(r_r, c_r) > U_{ss}(r_r, c_r)$
The safe groups prefer joint liability payment higher than \( \hat{c} \) and interest rates lower than \( \hat{r} \). Conversely, the risky groups prefer joint liability payments lower than \( \hat{c} \) and interest rate higher than \( \hat{r} \). With joint liability payment, the lender is able to charge each type a different interest rate. The lender can tailor his contract for the borrower depending on her type. This allows the lender to get back to the first best world where each type was charged a different interest rate.

### 2.2.1 Optimal Contracts

There are potentially two types of optimal contract. The separating contracts were the safe group’s contract is northeast of \((\hat{c}, \hat{r})\) and the risky group’s contract which is southeast of the this point. The second kind of contract is the pooling contract at \((\hat{c}, \hat{r})\).

### 2.2.2 Solving the Under-investment Problem

Under-investment takes place in the individual lending when

\[
\rho + u < \tilde{x} < \frac{pr}{p} \rho + u.
\]

The safe type are not lent to even though their projects are socially productive. With joint liability separating contracts (above), the safe type are lent to if the
following condition is met:

$$\bar{x} > \left( \frac{p_s + p_r}{p_r} \right) \rho$$

This condition just ensures that the LLC is to the right of \((\hat{c}, \hat{r})\). That is \(\hat{R} \geq \hat{c} + \hat{r}\). With the pooling contracts explained above, the safe type are lent to if the following condition is met:

$$\bar{x} > \left( \frac{p_s}{p_r} \right) \rho + \beta u$$

where \(\beta \equiv \theta p_r^2 + (1 - \theta)p_r^2_s\).

This condition ensures that the limited liability constraint is satisfied for the joint liability contract.

### 2.2.3 Solving the Over-investment Problem

Over-investment takes place in the individual lending when

$$\frac{p_r}{\rho} p_{\rho} + u < p_r x < \rho + u.$$  

The risky type are lent to even though their projects are socially unproductive. In group lending, the risky types participation constraint when she is paired up with another risky type would be given by:

$$p_r x - [p_r r + p_r (1 - p_r)c] \geq u$$

The lender’s zero profit constraint for the risky groups is given by

$$p_r r + p_r (1 - p_r)c = \rho$$

This implies that the risky type’s participation constraint would be satisfied if

$$p_r x \geq \rho + u$$

This eliminates the over-investment problem. The risky borrowers with the socially unproductive projects will drop out on their own. The condition below ensures that \((\hat{c}, \hat{r})\) satisfies the limited liability constraint.

$$x > \left( \frac{1}{p_s} + \frac{1}{p_r} \right) \rho$$
Lecture 3

Moral Hazard

In this lecture we examine the two approaches to the moral hazard problem taken in the literature. The first kind is the project choice model, where the borrower chooses between a risky and safe project. The second kind of model is the effort choice model, where the borrower chooses whether to exert high or low effort on her project. We also link in the role of monitoring, and show how it can play an extremely important role in alleviating the moral hazard problem.

3.1 Project Choice Model

In this section we explore the moral hazard problem associated with choosing the right kind of project. Stiglitz (1990) made seminal early contribution to the literature and explored project choice. I have set up a simple model in this section which is able to convey the salient points from Stiglitz (1990).

The borrowers are wealthless and aspire to borrow funds from the lender to invest into the projects. The projects produce positive output when it succeeds and 0 output when it fails. The borrower has the option of undertaking either a risky project or a safe project. The respective projects succeed with the probability $p_r$ and $p_s$ with $p_r < p_s$.

Even though the risky project requires a fixed initial sunk-cost investment of $\alpha$, it compensates by giving a higher marginal return to scale $\beta_r$ than the safe project $\beta_s$. Conversely, the safe project has no initial fixed cost investment and has a lower marginal return to scale.
3.1.1 Individual Lending

The lender cannot observe the project undertaken and thus has to influence the project choice through the contract he offers the borrower. The lender specifies the size $L$ and rate of interest $r$ of the loan. The lender’s own opportunity cost of capital is $\rho$ and the loan market is competitive, which ensures that the lender makes zero profits. Lender’s zero profit condition is given below.

$$ r = \frac{\rho}{p} \quad i = s, f $$ (L-ZPC)

The types of projects are summarised in table 3.1.1. We assume that the risky project has a higher expected marginal return to scale than safe project.

**Assumption 1.** $p_r \beta_r - p_s \beta_s = k$

That is the expected marginal return on scale is $k$ amount higher for the risky project as compared to the safe project. Thus, the borrower compares the

![Figure 3.1: Safe and Risky Projects](image_url)

higher expected marginal return (net of the interest rate payments) with the

<table>
<thead>
<tr>
<th>Project</th>
<th>Successful</th>
<th>Failure</th>
<th>Investment</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prob. Output</td>
<td>Prob. Output</td>
<td>Sunk-Cost</td>
<td>Scale</td>
</tr>
<tr>
<td>Risky</td>
<td>$p_r$ $\beta_r L$</td>
<td>$1 - p_r$</td>
<td>$0$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Safe</td>
<td>$p_s$ $\beta_s L$</td>
<td>$1 - p_s$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>
sunk cost when he decide between the risky and the safe project.

\[ V_r > V_s \]
\[ p_r(\beta_r L - rL) - \alpha > p_s(\beta_s L - rL) \]
\[ L > \frac{\alpha}{\Delta p r + k} \]

(3.1)

At a given interest rate, the borrower prefers undertaking a risky project beyond the threshold defined by (3.1). This threshold is reached when the higher expected marginal return \(^1\) of the risky project overwhelms the initial fixed cost investment associated with it.\(^2\) With a higher interest rate, the difference between the two projects expected marginal return to scale decreases and leading to decreases in the value of the threshold.

Figure 3.2: Switch Line and Optimal Contract under Individual Lending

In the \(L - r\) space, we can draw the locus of \(r\) and \(L\), where the borrower is indifferent between undertaking a risky or a safe project. This line is the threshold level of scale beyond which the borrower prefers undertaking a risky project. The line would have a negative slope to reflect the fact that higher

---

\(^1\)net of interest rate

\(^2\)By choosing the risky project, the borrower gains are an increase in expected marginal return of \(kL\) and lower expected interest rate payment \(\Delta prL\). She also loses the sunk cost investment of \(\alpha\). The threshold scale is the one which balances the two and makes the borrower indifferent between the two types of projects.
interest rate lower the threshold scale.\(^3\)

\[
L = \frac{\alpha}{\Delta p r + k} \quad (3.2)
\]

Using lender’s zero profit condition (L-ZPC) for safe projects and (3.2), we can find the range of contracts which are able to induce the borrower to choose a safe project over a risky one.

For the safe projects, the lender should be charging \(\frac{\rho p_s}{\rho s}\) interest rate. This is the risk-adjusted interest rate using (L-ZPC). At interest rate \(\frac{\rho p_s}{\rho s}\), \(L^*\), the maximum loan size is given by\(^4\)

\[
L^* = \frac{\alpha}{\Delta p^* \rho s + k}
\]

If he lender lends more than that, the borrower would automatically switch of a risky project.

**Group Lending**

In group lending the lender lends to groups of two. The additional repayment requirement in group lending is the joint liability payment \(c\). This is incurred if the borrower succeeds but her peer fails. Thus, for a group undertaking identical projects of the type \(i\) the probability with which a particular lender incurs the joint liability payment is given by \(p_i(1 - p_i)\).\(^5\) Borrower’s payoffs under group lending with joint liability payment is given by.

\[
V_{ss} = p_s(\beta_s L - rL) - p_s(1 - p_s)cL
\]
\[
V_{rr} = p_r(\beta_r L - rL) - \alpha - p_r(1 - p_r)cL
\]

where \(V_{ss}\) and \(V_{rr}\) are the borrower’s payoffs respectively when the groups symmetrically undertake risky and safe projects. Even though at first glance it may seem that the payoffs are lowered due to the joint liability payment, it turns out the group lending increases the loans size which in turn increases their payoffs.

Again, we are looking for the new switch, which gives us the locus of the

\(^3\)The switch line can also be written as \(r = \frac{1}{\rho s} (\frac{\alpha}{\rho s} - k)\), which could be interpreted as the highest interest rate the lender can charge on a loan of size \(L\) before the borrower switches to the risky projects.

\(^4\)We find this using the (L-ZPC) and (3.2)

\(^5\)We assume that the borrowers in a group make their decision cooperatively and after full communication. They also have perfect information about each other. This allows us to restrict our analysis to the symmetric choices where either both the borrowers undertake risky projects or both undertake safe projects. If the borrowers had imperfect information about each other, they interact strategically with each other and the analysis can no longer be restricted to symmetric decisions.
contracts where the borrower is indifferent between undertaking the risky or the safe project. The borrower would undertake a risky project if the following condition is met.

\[ V_{rr} > V_{ss} \]

\[ p_r(\beta_r L - rL) - \alpha - p_r(1 - p_r)cL > p_s(\beta_s L - rL) - p_s(1 - p_s)cL \]

This gives us the threshold loan size beyond which the borrower would undertake a risky project.

\[ L > \frac{\alpha}{\Delta p r + k - \Delta p(p_s + p_r - 1)c} \]  \hspace{1cm} (3.3)

consequently, at a given interest rate \( r \) and joint liability payment \( q \), the borrower prefers undertaking a risky project beyond the threshold loan size defined by (3.3).

We now need incorporate the joint liability payment \( c \) in the lender’s zero profit condition. For a group undertaking project of the type \( i \), the lender receives \( c \) with the probability \( p_i(1 - p_i) \), when one member of the group succeeds and one member of the group fails. As the lender shifts the repayment burden to the peer by increase \( c \), the interest fall concomitantly. If the lender is lending to group that undertakes a safe projects, his zero profit condition would be as follows.

\[ p_s r + p_s(1 - p_s)c = \rho \]

\[ r = \left( \frac{\rho}{p_s} \right) - \left( \frac{1 - p_s}{p_s} \right)c \]  \hspace{1cm} (L-ZPC(G))

Thus, due to joint liability payment \( c \), the interest rate charged in group lending is lowered by amount \( \left( \frac{1 - p_s}{p_s} \right)c \) as compared the interest rate in individual lending. Using the interest rate and the threshold level defined by (3.3), we can find the maximum loan size the lender would be willing to give the borrower in group lending. Given the opportunity cost of capital \( \rho \), the maximum loan size is given by the following expression.

\[ L^*_{G} = \frac{\alpha}{\Delta \rho \left( \frac{p_s}{p_r} \right) + k - \varphi c} \]  \hspace{1cm} (3.4)

where \( \varphi = \Delta \rho \left( \frac{1 - p_s}{p_s} + (p_s + p_r - 1) \right) \). \( \varphi > 0 \) if \( p_s + p_r > 1 \). It should be clear from (3.4) that for \( c > 0 \), the borrower obtains a larger loan in group lending than in individual lending. Further, as \( c \) increases, the loan size increases. Undertaking some
burden of repayment in case of the peer’s failure thus allows the borrower to get loans of larger size in group lending.

\[ \rho \frac{\Delta p}{p_s} - (1-p_s)k \]

Figure 3.3: Switch Line and Optimal Contract under Group Lending

### 3.2 Effort Choice Model

This section is based simple versions of the models in Aniket (2006) and Conning (2000). A project requires an investment of 1 unit of capital and produces output \( x \) with probability \( \pi \) and 0 with probability \( 1-p \), where \( i \) is the effort level exerted by the borrower.\(^7\) If the borrower is diligent and exerts high effort level (\( i = h \)) the project succeeds with probability \( \pi^h \). Conversely, if the borrower exerts low effort (\( i = l \)) the project succeeds with probability \( \pi^l \) and the borrower enjoys private benefits \( B \). These private benefits are are only visible to her and not to other borrowers or lenders.\(^8\) We assume that the borrower’s reservation utility is 0.

\(^7\)Note that we have chosen to use \( p \) to represent probability associated with the inherent characteristics of either the project or a borrower and \( \pi \) with effort which the borrower may choose explicitly.

\(^8\)We assume in latter sections that other borrowers may curtail the value of these private benefits enjoyed by a borrower if they bear some cost to themselves. The lender is not able to curtail these private benefits at all.

\(^9\)An alternative way of looking at this would have been to assume that exerting high effort is more costly for the borrower as compared to the low effort.
3.2.1 Perfect Information Benchmark

In the perfect information world, we can observe the borrower’s effort level and ensure that she exerts a high effort level. He can thus offer her a contract contingent on her effort level. The constraints the optimal contract needs to satisfy are the borrower’s participation and limited liability constraint and the lender’s break even condition.

We assume that the borrower is wealth-less and thus the limited liability constraint applies. The limited liability constraint just says that the borrower cannot pay more than the output of the project. This just implies that the borrower’s interest rate should be greater than $x$ and should be allowed to default in case the project fails.

The borrower’s participation constraint is satisfied if the borrower has sufficient incentive to accept the contract. If the project succeeds, the borrower pays an interest rate of $r$ on the loan. If it fails, the borrower declares default and pays nothing. Given borrower’s effort level $i \in \{h, l\}$, her expected payoff is given by $\pi^i(x - r)$.

$$\pi^h(x - r) \geq 0 \quad \text{(PC-I)}$$

If the $x \geq r$, then the participation constraint would be satisfied. In the perfect information world, the lender is able to ensure that the borrower exerts high effort. The lender’s break even constraint requires that his profits are non-negative are would be as follows.

$$\pi^h r \geq \rho \quad \text{(L-ZPC-I)}$$

Lender’s break even constraint is satisfied if $r \geq \frac{\rho}{\pi^h}$. We find that the participation constraint puts an upper bound on the interest rate and the break even constraint puts a lower bound on the interest rate. Thus, for an optimal contract, the interest rate has to be in the following range.

$$\frac{\rho}{\pi^h} \leq r \leq x \quad (3.5)$$

The first thing to notice about (3.5) is that an optimal contract and thus a feasible interest rate would exist only if the $\frac{\rho}{\pi^h} \geq x$. That is, if the project is not more productive than the opportunity cost of capital, it would not be financed even in the first best world. Put another way, the project should be socially viable.

---

10 Turns out that the limited liability constraint and the participation constraint are identical in this case.
Now let's assume that the project is strictly socially viable, i.e., $\frac{\rho}{\pi} < x$. Then $r$ can take any value in the range $(\frac{\rho}{\pi}, x)$. If $r = \frac{\rho}{\pi}$, then the borrower's expected payoff is $\pi h (x - \frac{\rho}{\pi})$ and the lender makes zero profit.\(^{11}\) Conversely, if $r = x$, then the borrower's expected payoff is 0 and the lender makes expected profits of $\pi h x - \rho$.\(^{12}\)

What this shows us is that financing a socially viable project creates a positive surplus, $\pi h x - \rho$ in this case. This can either be allocated entirely to the borrower or entirely to the lender or shared between the two.

*Lender’s Break Even versus Zero Profit Condition:* Who gets what proportion of the profit depends entirely on the relative bargaining position of the borrower and the lender. If the lender has all the bargaining position, he would keep the entire surplus. This is the case if the lender was a monopolist.\(^{13}\) Conversely, if there is a competitive loan market, the lender would be undercut by his competitors till he makes zero profit. In this case the lender has no relative bargaining strength and all the bargaining power lies in the hand of the borrower. We have been referring to this case as the *zero profit condition*.

Now let's deviate for a moment and think of how a higher borrower's reservation utility\(^{14}\) would change the analysis. If the borrower reservation utility $u$ increases, the surplus created is decreased. Who gets the surplus still gets determined by the relative bargaining strength.

Solving any optimal contract problem entails finding the *contract space* or the region which satisfies all the constraints and then using the objective function to find the optimal contract(s). In this case, with perfect information, the contract space is $r \in (\frac{\rho}{\pi}, x)$ and the objective function tells us whether we are maximising or minimising $r$. We maximise it if the lender is a monopolist and minimise it if the loan market is competitive.

### 3.2.2 Second Best World: Individual Lending

Let's analyse how the imperfect information changes the contract space. In the imperfect information world, the lender does not observe the borrower's effort level and have to induce the borrower to exert his proffered effort level (high in this case) through the contract he offers her. The incentive compatibility constraint below ensures that the borrower has sufficient incentive to exert high

---

\(^{11}\)The lender’s break even condition *binds* and the borrower’s participation constraint is *slack*.

\(^{12}\)which is positive because we assumed that $\frac{\rho}{\pi} < x$ as the beginning of this analysis.

\(^{13}\)In this case, we maximise the lender’s profit subject to his *break even condition*.

\(^{14}\)We have assumed that his 0 till now.
effort.

\[ \pi^h(x - r) \geq \pi^l(x - r) + B \]  \hspace{1cm} (ICC-I)

\[ r \leq x - \frac{B}{\Delta \pi} \]

The participation constraint puts an upper bound on \( r \). If the interest rate is too high, it interferes with the borrower’s incentive to exert high effort. The contract space is the range of \( r \) which satisfies the borrower’s participation and incentive compatibility constraint and the lender’s break even condition. The borrower’s participation constraint and the lender’s break even constraint is identical to the ones given by (PC-I) and (L-ZPC-I). The lender’s break-even constraint puts an lower bound on the interest rate and the borrower’s participation constraint puts an upper bound on the interest rate.\(^{15}\) All three constraints above can be satisfied if the following conditions are met.

\[ \frac{\rho}{\pi^h} \leq r \leq \left( x - \frac{B}{\Delta \pi} \right) \]  \hspace{1cm} (3.6)

Comparing the (3.6) to the (3.5), we find that the range is curtailed in the second best world due to the incentive compatibility constraint. If the interest rate is set in the range \( \left( \frac{\rho}{\pi^h}, x - \frac{B}{\Delta \pi} \right) \), then the borrower would definitely exert high effort.

In the first best world, allocating the borrower 0 expected payoff satisfied her participation constraint. In the second best world, 0 expected payoff does not satisfy the participation constraint and thus the lender has to offer her expected payoff of at least \( \pi^h \left( \frac{B}{\Delta \pi} \right) \) to ensure that she exerts high effort.\(^{16}\)

In the first best world, the surplus created by financing the project is \( \pi^h x - \rho \). In the first best world, this was shared amongst the borrower and the lender according to the relative bargaining strength. Imperfect information reduces the surplus by \( \pi^h \frac{B}{\Delta \pi} \), the rent allocated to the borrower in order to incentivise her

\(^{15}\)It should be clear that the incentive capability constraint puts puts a smaller upper bound on the \( r \) than the participation constraint and thus we can ignore it. If the borrower’s incentive compatibility constraint binds, then her participation constraint would automatically be satisfied.

\(^{16}\)If (ICC-I) holds with equality, it gives us \( x - r = \frac{B}{\Delta \pi} \) which implies that the borrower’s expected payoff should be \( \pi^h \frac{B}{\Delta \pi} \) at the least.
to exert high effort. In the second best world, the surplus created by financing the project is $\pi^h (x - \frac{B}{\Delta \pi}) - \rho$,\(^{17}\) which is shared between the borrower and the lender according to the relative bargaining strength.

**Lending Efficiency:** This is connected to the concept of lending efficiency. The first best world surplus of a project is reduced by the rent allocated to agents by the principal to incentivize them to take a particular action. For every institutional mechanism, we can find the associated surplus. The lower the rents allocated to the borrowers, the higher the surplus created by the project. Thus, the lower the rents required to implement a project (in this case, to get it financed) the more efficient the project is considered. Lending efficiency is thus the metric by which all the institutional mechanism are evaluated.

**Borrower’s Private Benefits:** It should be obvious that anything that decreases the borrower’s private benefit $B$ should be able to increases the surplus from the project and thus increase the lending efficiency. There are a category of models that look at how efficiently monitoring can reduce the private benefits and increase the lending efficiency.

### 3.2.3 Delegated Monitoring

The lender has no ability to reduce the borrower’s private benefits but he could hire someone who lives in the same area or is socially connected to the borrower to do exactly that. Let assume that this person is able to reduce the private benefits of the borrower by monitoring her. Specifically, the borrower’s private benefit $B$ is a function how intensively monitor monitors her. To the monitor the cost of monitoring is $m$. As $m$ increases, the monitor monitors more intensely and $B$, the private benefits fall. Assumption below characterise the monitoring function.

**Assumption 2** (Monitoring Function $B(m)$): $B(m) > 0$; $B'(m) < 0$.

Of course, now the lender would have to incentivize the monitor by making her payoff contingent on the outcome of the project.\(^{18}\) Incentivizing the monitor would require satisfying her limited liability, participation and incentive compatibility constraint. We assume that like the borrower, the monitor’s reservation utility is 0. The limited liability constraint ensures that the monitor’s wage $w$

---

\(^{17}\)This distance of the green arrow in Figure 3.2.2 multiplied by the probability of success.

\(^{18}\)Since that is the only signal the lender gets, he has no option but to make the monitoring payoff contingent on that signal.
is not less than 0 irrespective of the project outcome.\(^{19}\)

\[
\begin{align*}
\pi^h w &\geq 0 \\
\pi^h w - m &\geq \pi^l w
\end{align*}
\] (PC-M) (ICC-M)

The participation constraint is satisfied for any non-negative \(w\). The incentive compatibility condition is satisfied if \(w \geq \frac{\pi^l}{\pi^h}\). So, the cost of getting \(m\) amount of monitoring for the lender is offering the monitor a wage of at least \(\frac{\pi^l}{\pi^h}\) if the project succeeds. In expected terms, this cost is at least \(\pi^h \frac{\pi^l}{\pi^h}\). We can see the examine the benefits if we look at the borrower’s rent this has reduced.

\[
\pi^h (x - r) \geq \pi^l (x - r) + B(m)
\] (ICC-I’)

The borrower’s payoff if the project succeeds is \(B(m)\), which is less than payoff the borrower got when there was no monitoring. With monitoring the expected surplus of the project is \(\pi^h \left( x - \frac{B(m)}{\Delta \pi} - \frac{m}{\Delta \pi} \right) - \rho\).

The optimal amount of monitoring is the \(m\) that maximises the surplus. That is \(B'(m) = -1\). Thus, there would be positive amounts of monitoring if \(B'(0) < -1\). Further, if this condition holds, it should be clear that the lending efficiency has increased with monitoring.

### 3.2.4 Simultaneous Group Lending

In this section we examine the lending efficiency of group lending under costly monitoring described by Assumption 2. In simultaneous group lending, borrower form into groups of two before they approach the lender for a loan. The lender offers the group a contract contingent on the state of the world, i.e., the outcome of the project. Without loss of generality, we can confine ourselves to the contract where each borrowers are obliged to pay interest rate \(r\) on their loans if both the projects succeed and 0 if both project fails. If only one of the two projects succeeds, joint liability kicks in and the lender confiscates the full output \(x\) of the project. To summarise, the borrowers get a positive payoff only when the both projects succeed. In all other cases, they get a 0 payoff.

If they accept the contract offered by the lender, the first decide on the intensity with which they would monitor each other and subsequently choose the effort level. Once the outcome is realised, the borrower get their payoff depending on the outcome of the project. The contract space is determined by the following two constraints.\(^{20}\)

\(^{19}\)This just means that the lender cannot penalise the monitoring for the failure of the project.

\(^{20}\)See Aniket (2006, Pages 30-33)
1. The individual borrower’s incentive compatibility condition in group lending (ICC-Sim) which ensures that the borrower exerts high effort when her peer exerts high effort \((j = h)\) and both choose to monitor with intensity \(m\).

\[
(\pi^h)^2(x - r) - m \geq \pi^h \pi^l (x - r) + B(m) - m \tag{ICC-Sim}
\]

\[
r \leq x - \left( \frac{B(m)}{\pi^h \Delta \pi} \right)
\]

2. The group’s collective compatibility condition (GCC).

\[
(\pi^h)^2(x - r) - m \geq (\pi^l)^2(x - r) \tag{GCC}
\]

\[
r \leq x - \left( \frac{B(0) + m}{(\pi^h + \pi^l)\Delta \pi} \right)
\]

(ICC-Sim) and (GCC) can be summarised in the following condition:

\[
r \leq x - \frac{1}{\pi^h \Delta \pi} \max \left( B(m), \alpha(B(0) + m) \right)
\]

where \(\alpha = \frac{\pi^h}{\pi^h + \pi^l}\). Again, the question is to find the optimal level of monitoring. The optimal level of monitoring would be the one which creates the greatest surplus, which would be achieved when \(\alpha(B(0) + m) = B(m)\). (H in Figure 3.4)

Assumption 2 ensures that there would be positive level of monitoring in group lending.
3.2.5 Sequential Group Lending

In sequential group lending, one borrower gets the loan while the second borrower is waiting for her loan. The second borrower only gets the loan if the first borrower succeeds. Again, borrowers only get a positive payoff if both borrowers borrow and the both projects succeed. Aniket (2006) shows that the both borrowers would choose to monitor with intensity \( m \) and exert high effort if the following condition is met:\(^{21}\)

\[
x \leq x - \frac{1}{\pi^h \Delta \pi} \max \left(B(m), m\right)
\]

The surplus would be maximised and the optimal level of monitoring would be achieved when \( B(m) = m \). (G in Figure 3.4) Looking at Figure 3.4, it should be clear that sequential lending creates a greater surplus as than simultaneous lending. This is because in simultaneous lending, the group’s collective incentive compatibility conditions (GCC) has to be satisfied. This is akin to the group behaving cooperatively just like it was able to do in Stiglitz (1990). Even though a group behaving cooperatively does better than individual lending, it is not much of an improvement in a multi-tasking environment, i.e., the two-task environment in Aniket (2006) where the lender has to incentivise monitoring and effort level. In a two-task environment, the sequential lending does much better because the lender has to incentivise the tasks individually (ICC-Seq) and not collectively (GCC).

\[\text{Figure 3.5: Monitoring Intensities as Monitoring Efficiency Increases}\]

Let’s now examine what happens if we vary the monitoring function. Let's

\(^{21}\)See Aniket (2006, Pages 33-36)
think of a parameter $\beta$ that controls the efficiency of the monitoring function. With a higher $\beta$ increases, a given $m$ is associated with a lower $B$. Figure 3.5 shows how the monitoring function moves towards the origin with as $\beta$ increases. What is interesting is that as $\beta \Rightarrow \infty$, the monitoring becomes more and more efficient and we get closer to the first best world or to almost perfect information. With $\beta \Rightarrow \infty$, the borrowers are still allocated a positive payoff in the simultaneous lending where as in sequential lending they are allocated 0 payoffs. That is even with almost perfect information, sequential group lending can achieve first best where as simultaneous group lending cannot.\textsuperscript{22}

\begin{footnotesize}
\textsuperscript{22}With almost perfect information, the contract space for simultaneous group lending is $\frac{\rho}{\pi} \leq r \leq x - \alpha B(0)$ and sequential group lending is $\frac{\rho}{\pi} \leq r \leq x$.
\end{footnotesize}
Lecture 4

Costly State Verification

Costly state verification and enforcement are closely tied together. Both discourage the borrower from strategically defaulting or lying about the state of the project upon its completion. When the lender depends entirely on brute enforcement, he does not know the actual state of the project and thus imposes punishment blindly on any borrower that declares default. One can see how this may be deemed unfair and unjust by the borrowers, especially if the probability of project failure is very high. Conversely, the lender can undertake auditing or verify the state to find out the state of the project and punish accordingly. In this case, the lender would punish only when the borrower is lying about the state of project.

The models of costly state verification analyse the determinants of the lender’s decision to audit when auditing the project is costly in terms of resources. The lender’s objective is to audit with sufficient frequency to discourage the borrower from lying.

Individual Lending

This section is based on the costly state verification model in Ghatak and Guinnane (1999).

The project undertaken by the borrower requires 1 unit of capital and produces high output $x$ with probability $p$ and low output 0 with probability $(1 - p)$.

If the borrower declares default due to low output, the lender cannot be

\[^1\text{The terms auditing and state verification are used interchangeably. Though, auditing does not tell us if the process is costless or costly.}\]
certain whether the borrower has defaulted involuntarily or strategically.\textsuperscript{2} To discourage strategic default, the lender would like to verify that state of the project with a positive probability. If the lender verifies with sufficient frequency, the borrower would prefer to tell the truth.

If the lender could verify the state of the project costlessly, the lender would do so all the time and the borrower would never lie about the state of her project. Conversely, if state verification is costly, the borrower may declare default hoping that lender would not auditing the project. The lender would offer the borrower a contract that specifies the probability of audit conditional on the borrower declaring default. This would give the borrower the incentive to report the truthful state of the project.

We make the reasonable assumption that the lender only audits when the borrower declares default. The lender’s cost of verifying the state of the project is $\kappa$. If the lender finds that the borrower is lying, he confiscates the project output. The lender opportunity cost of capital is $\rho$ and the borrower’s reservation wage is 0.

The Individual Lending Contract

The lender’s contract specifies $r$, the interest rate due at the completion of the project and $\lambda$, the probability with which the lender would audit the borrower’s project outcome if the borrower declares that she is defaulting due to low output.

The optimal contract $(r, \lambda)$ the satisfies the following constraints.

\begin{align*}
  p(x - r &> 0) \quad \text{(PC-I)} \\
  x - r &> (1 - \lambda)x \quad \text{(ICC-I)} \\
  pr - (1 - p)\lambda\kappa &> \rho \quad \text{(L-ZPC-I)}
\end{align*}

(PC-I) is the borrower’s participation constraint and (L-ZPC-I) is the lender’s zero profit condition under individual lending. (ICC-I) is the borrower incentive compatibility constraint or the truth-telling constraint. The left hand side is the payoff from repaying and the right hand side is the expected payoff from defaulting.\textsuperscript{3} It ensures that given $\lambda$, the borrower has no incentive to declare default when the output is high.

It should be clear under the optimal contract offered by the lender, (ICC-I) and (L-ZPC-I) would bind and (PC-I) would remain slack. We can rearrange

\textsuperscript{2}That the project has produced $x$ and the borrower wants to keep the whole project output to herself.

\textsuperscript{3}With probability $\lambda$ the lender would audit and take away the output and the with probability $1 - \lambda$ the borrower would be able to keep the output $x$. 

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(ICC-I) and (L-ZPC-I) and write them in the following way.

\[ r = x \cdot \lambda \]  \hspace{1cm} \text{(ICC-I')}

\[ r = \left[ \frac{\rho}{p} \right] + \left( \frac{1 - p}{p} \right) \kappa \cdot \lambda \]  \hspace{1cm} \text{(L-ZPC-I')}

Solving (ICC-I') and (L-ZPC-I') simultaneously gives us the value of \((r^*_I, \lambda^*_I)\).

\[ \lambda^*_I = \frac{\rho}{p} \frac{x}{p x - (1 - p) \kappa} \]

\[ r^*_I = x \lambda^*_I \]

**Figure 4.1: Costly State Verification in Individual Lending**

The interest rate charged in the optimal individual lending contract is given by \(r^*_I = \frac{\rho x}{p x - (1 - p) \kappa}\). The interest rate is increasing in \(\kappa\) and \(\rho\) and decreasing in \(x\).

**Group Lending**

We assume that there is no information problem between the borrowers in a group. The borrowers in the group can observe the state of each other’s project costlessly. We also assume that whether the lender is auditing one or two borrowers, the cost of auditing is the same. This assumption just implies once the
lender has incurred the fixed cost of auditing one borrower, the additional cost
of auditing the second borrower is negligible.

The Group Lending Contract

In group lending, the lender offer the group a contract where if the borrower
succeeds, she pays \( r \) her peer succeeds and pays \( 2r \) if her peer fails. If the
borrower fails, she pays nothing. Further, if the group declares default and fails
to pay \( 2r \) collectively, the lender audits the group at the cost of \( \kappa \).

\[
\begin{align*}
p(x - 2r) & \geq 0 \quad \text{(PC-G)} \\
x - 2r & \geq (1 - \lambda)x \quad \text{(ICC-G)} \\
p^2r + p(1 - p)2r - (1 - p)^2\lambda\kappa & \geq \rho \quad \text{(L-ZPC-G)}
\end{align*}
\]

(PG-G) and (ICC-G) is each borrower’s participation and incentive compati-
bility constraint and (L-ZPC-G) is the lender’s zero profit condition under group
lending.

The right hand side of (L-ZPC-G) gives us the lender’s opportunity cost of
1 unit of capital and the left hand side the returns from lending that 1 unit
of capital to the group. For each borrower that succeeds,\(^4\) the lender gets \( r \) if
her peer succeeds and gets \( 2r \) if her peer fails. If both fail, the lender spends \( \kappa \)
auditing the borrowers. The borrower’s two constraints take into account the
fact that she is required to pay interest of \( 2r \).

It should be clear under the optimal contract offered by the lender, (ICC-G)
and (L-ZPC-G) would bind and (PC-G) would remain slack. We can rearrange
(ICC-G) and (L-ZPC-G) and write them in the following way.

\[
\begin{align*}
r & = \frac{x}{2} \cdot \lambda \quad \text{(ICC-G’)} \\
r & = \frac{p}{p} \left[ \frac{1}{2 - p} \right] + \left( \frac{1 - p}{p} \right) \kappa \left( \frac{1 - p}{2 - p} \right) \cdot \lambda \quad \text{(L-ZPC-G’)}
\end{align*}
\]

Solving (ICC-G’) and (L-ZPC-G’) simultaneously gives use the value of \( (r^*_G, \lambda^*_G) \).

\(^4\)Note that with the optimal contract, the borrower’s always report the truth and have no
incentive to lie.
The interest rate charged in the optimal individual lending contract is given by $r_G^* = \frac{1}{2} x \lambda^*$. Thus, we find that $\lambda_G^* < \lambda_I^*$, the borrowers are audited with a lower probability and are consequently charged a lower interest rate, $r_G^* < r_I^*$.

With a lower probability of auditing, the resources absorbed in state verification decreases. If the lender is operating in a competitive market and has no bargaining strength, he would pass on the benefits of lower auditing cost to the borrower in form of lower interest rates. Conversely, if he has all the bargaining strength and is operating as a monopolist, he would retain all the benefits of lower auditing cost for himself. In either case, it is clear that the group lending is more efficient than individual lending. This is because group lending is able to ensure that the borrowers are truthful with a lower probability of auditing. With lower expected auditing costs, there is greater project surplus which is shared between the lender and the borrower according to their relative bargaining strength.
Rai and Sjöström (2004) analyse the role cross-reporting can play in the extracting the information on the state of the borrowers’ projects without undertaking the task of auditing. They assume that the borrower can perfectly observe each other outcome. Once the project is realise, the borrower make the decision on the repayment. Once they have made their decision they report it to the borrower along with information about the other borrower’s project. Rai and Sjöström (2004) show that the borrowers always report the truth about the other borrower. As a result the lender does not ever punish the borrower in equilibrium.
Bibliography


