

UNCONSTRAINED MAXIMISATION

Period 1 Income realisation received
Decision on c^1 period 1 consumption
 ... implying decision to **borrow** or **save** contingent

$$\tilde{c}_{bad}^1 \text{ solves } \max_{c_{bad}^1} u(c_{bad}^1) + E(u(c_{bad}^2))$$

$$\tilde{c}_{good}^1 \text{ solves } \max_{c_{good}^1} u(c_{good}^1) + E(u(c_{good}^2))$$

Period 2 Income realisation received
repay or **receive repayment**
 c^2 the residual income is consumed in period 2

CONSTRAINED MAXIMISATION

Individual credit ceiling is B
 Upper bound of relevant rate of B
 If bad state is realised in period 1, the $c_{bad}^*(B)$ solves:

$$\max_{c^1} u(c_{bad}^1) + E(u(c_{bad}^2))$$

subject to $b \leq B$.

$$c_{bad}^*(B) = \begin{cases} (z - \sigma) + B & \text{for } B < B_c \\ \tilde{c}_{bad}^1 & \text{for } B \geq B_c \end{cases}$$

If good state is realised in period 1,
 \tilde{c}_{good}^1 solves $\max_{c_{good}^1} u(c_{good}^1) + E(u(c_{good}^2))$

UN/CONSTRAINED MAXIMISATION

		Consumption	
		Period 1	Period 2
States in Period 1	bad	\tilde{c}_{bad}^1	Total income - \tilde{c}_{bad}^1
	good	\tilde{c}_{good}^1	Total income - \tilde{c}_{good}^1

Table: Agent's consumption in all states without credit ceiling

		Consumption	
		Period 1	Period 2
States in Period 1	bad	$c_{bad}^*(B)$	Total income - $c_{bad}^*(B)$
	good	\tilde{c}_{good}^1	Total income - \tilde{c}_{good}^1

Table: Agent's consumption in all states with a binding credit ceiling B

RISK PREMIUM

Expected Utility depends on
 z (expected income)
 σ (volatility of income)
 B (credit ceiling) if $B \leq B_c$
Expected utility is increasing and concave in B till B_c and flat beyond

Certainty equivalent income x is the risk-less income = expected utility from the uncertain income process

$$2U(x) = EU(B, z, \sigma)$$

where $x = z - \pi_{risk}$

x can be broken down into expected income z and risk premium π_{risk} .

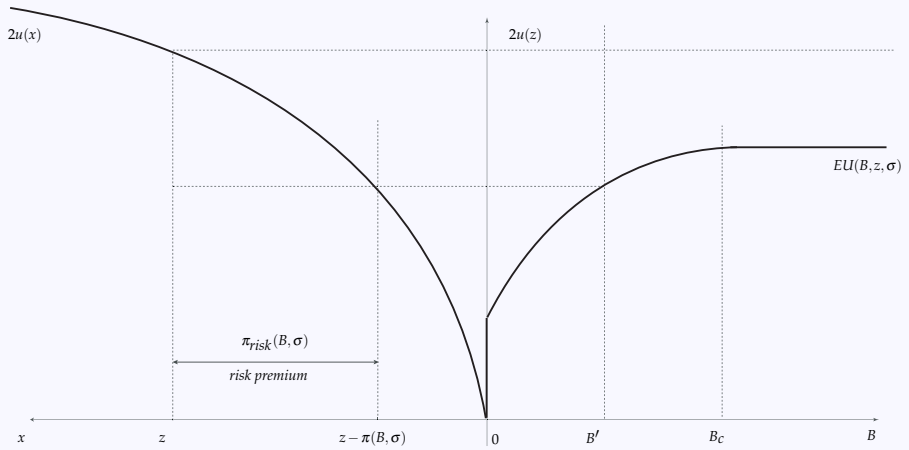


Figure: Obtaining the Risk Premium
 Certainty Equivalent Income = Expected Utility

BORROWER'S PROJECT & TYPE

- Borrower's project

$$1 \text{ unit of capital} \rightarrow \begin{cases} x_i & \text{with probability } p_i \\ 0 & \dots (1 - p_i) \end{cases}$$

- Borrower type $i = \{s, f\}$

$$\begin{cases} p_s & \text{(Safe type)} \\ p_r & \text{(Risky type)} \dots p_r < p_s \end{cases}$$

- Borrower's type unobservable to lender

ENVIRONMENT

- Impoverished borrower i
 - Risk neutral
 - No wealth
 - Reservation utility is \bar{u}
 - proportion of type $r \rightarrow \theta$
 - ... type $s \rightarrow 1 - \theta$
- Lender
 - Risk neutral
 - opportunity cost of capital ρ
 - Lends in a competitive loan market

FIRST BEST: PERFECT INFORMATION BENCHMARK

- If the lender knows borrower's type (perfect information environment) then the lender's profit condition would be:

$$r_i = \frac{\rho}{p_i} \quad i = r, s \quad (\text{L-ZPC})$$

... lender charges r and s different rate
 ... risky type pays a higher interest rate

- Borrower i 's expected payoff

$$U_i(r) = p_i(x_i - r_i)$$

Recall that the borrower is risk neutral and thus only cares about her expected payoff.

POSITIVE ASSORTATIVE MATCHING

Proposition (Positive Assortative Matching)

Joint Liability contracts lead to positive assortative matching.

$$U_{ij}(r, c) = p_i p_j (x_i - r) + p_i (1 - p_j) (x_i - r - c)$$

$$= p_i (x_i - r) - p_i (1 - p_j) c$$

$$U_{rs}(r, c) - U_{rr}(r, c) = p_r (p_s - p_r) c \tag{1}$$

$$U_{ss}(r, c) - U_{sr}(r, c) = p_s (p_s - p_r) c \tag{2}$$

$$(2) > (1)$$

POSITIVE ASSORTATIVE MATCHING AND SOCIAL OPTIMUM

Paper (Ghatak, 1999, 2000)

Joint Liability Group Lending leads to *positive assortative matching* solves the problems of *under* and *over-investment*.

Assumption (Socially Optimal Matching)

Positive assortative matching maximises the aggregate expected payoffs of borrowers over all possible matches

$$U_{ss}(r, c) - U_{sr}(r, c) > U_{rs}(r, c) - U_{rr}(r, c) \tag{(2) > (1)}$$

$$U_{ss}(r, c) + U_{rr}(r, c) > U_{rs}(r, c) + U_{sr}(r, c) \tag{(rearranging)}$$

INDIFFERENCE CURVES

Indifference Curve of borrower type *i*

$$U_{ij}(r, c) = p_i (x_i - r) - p_i (1 - p_j) c = \bar{k}$$

$$\left[\frac{dc}{dr} \right]_{U_{ij}=\text{constant}} = - \frac{1}{1 - p_j}$$

s type's indifference curve steeper

$$\left| - \frac{1}{1 - p_s} \right| > \left| - \frac{1}{1 - p_r} \right|$$

INDIFFERENCE CURVES OF THE TWO TYPES

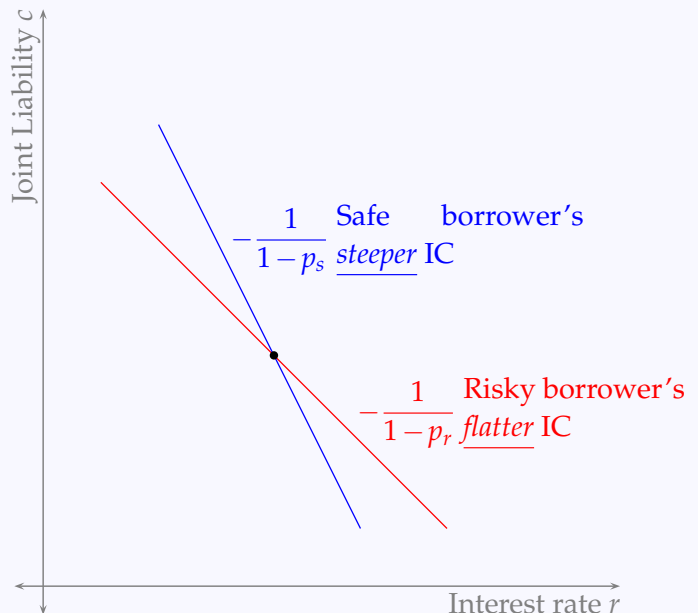


Figure: Risky and Safe Types' Indifference Curves

LENDER'S PROBLEM

- Lender offers group contracts (r_r, c_r) and (r_s, c_s) which maximise the borrower's payoff subject to the following constraint's:

$$r_r p_r + c_r (1 - p_r) p_r \geq \rho \Rightarrow \frac{dc}{dr} = -\frac{1}{1 - p_r} \quad (\text{L-ZPC}_r)$$

$$r_s p_s + c_s (1 - p_s) p_s \geq \rho \Rightarrow \frac{dc}{dr} = -\frac{1}{1 - p_s} \quad (\text{L-ZPC}_s)$$

$$U_{ii}(r_i, c_i) \geq \bar{u}, \quad i = r, s \quad (\text{PC}_i)$$

$$x_i \geq r_i + c_i \quad i = r, s \quad (\text{LLC}_i)$$

$$U_{rr}(r_r, c_r) \geq U_{rr}(r_s, c_s) \quad (\text{ICC}_{rr})$$

$$U_{ss}(r_s, c_s) \geq U_{ss}(r_r, c_r) \quad (\text{ICC}_{ss})$$

ABBREVIATIONS

L-ZPC_i Lender's Zero Profit Condition for type *i*

PC_i Participation Constraint for type *i*

LLC_i Limited Liability Constraint for type *i*

ICC_{ii} Incentive Compatibility Constraint for group *i, i*

SEPARATING EQUILIBRIUM IN GROUP LENDING

- (L-ZPC_s) and (L-ZPC_r) cross at (\hat{r}, \hat{c})

Proposition (Separating Equilibrium)

For any joint liability contract (r, c)

- if $r_s < \hat{r}, c_s > \hat{c}$, then $U_{ss}(r_s, c_s) > U_{rr}(r_s, c_s)$
- if $r_r > \hat{r}, c_r < \hat{c}$, then $U_{rr}(r_r, c_r) > U_{ss}(r_r, c_r)$

- Safe groups prefer high joint liability payment low interest rates
- Risky groups prefer low joint liability payments high interest rate
- Different interest rates for different types – back to the perfect information environment

SEPARATING EQUILIBRIUM IN r - c SPACE

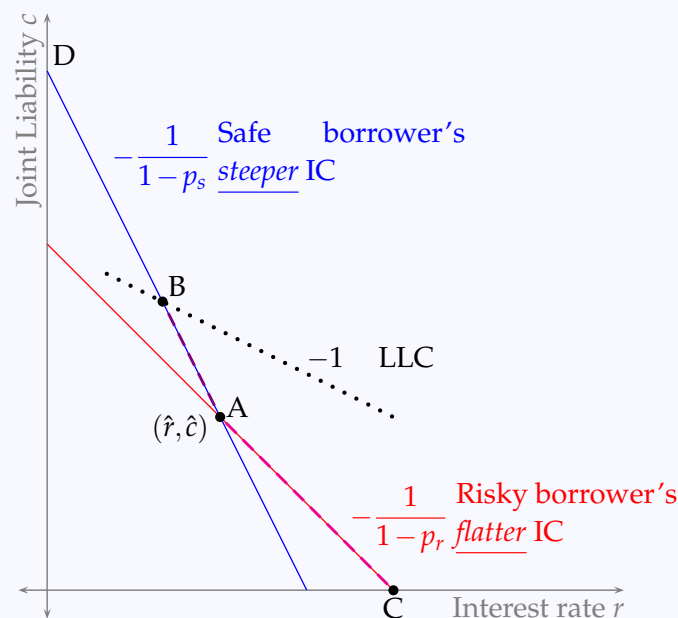


Figure: Separating Joint Liability Contract

CONTRACTS

Separating Contract

- Safe: Segment BA
- Risky: Segment AC

Conditions: Projects sufficiently productive to satisfy the Limited Liability Condition (LLC) along respective contract segments.

Under-investment:

Bring back the safe borrowers with socially productive investment.

Over-investment:

Risky borrowers with socially productive investment drop out.

Pooling Contract

- (\hat{c}, \hat{r}) at A

MORAL HAZARD: PROJECT CHOICE MODEL – STGLITZ (1990)

Borrowers

- Risk neutral
- Wealth-less
- Choose between **safe** and **risky** project

Project	Successful		Failure		Investment		Interest
	Prob.	Output	Prob.	Output	Sunk-Cost	Scale	
<i>Risky</i>	p_r	$\beta_r L$	$1 - p_r$	0	α	L	rL
<i>Safe</i>	p_s	$\beta_s L$	$1 - p_s$	0	0	L	rL

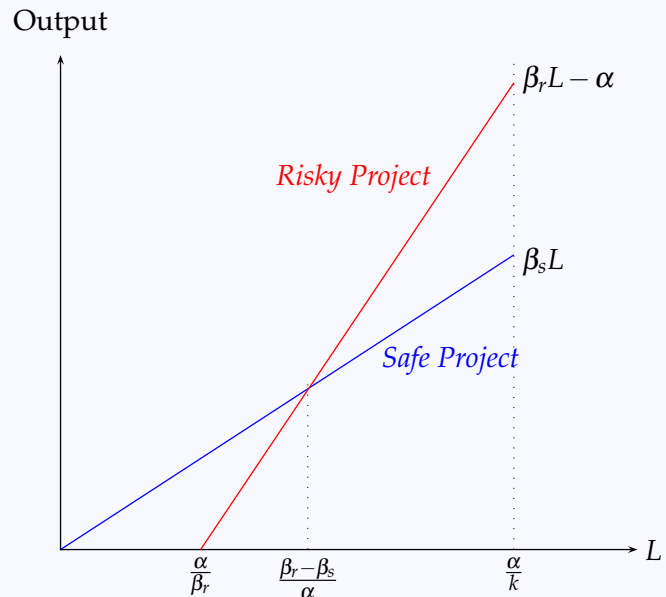


Figure: Safe and Risky Projects

BORROWER'S PAYOFF FROM THE TWO PROJECTS

Safe Project: Lower expected marginal return & 0 sunk cost

$$V_s = p_s(\beta_s L - rL)$$

Risky Project: Higher expected marginal return & α sunk cost

$$V_r = p_r(\beta_r L - rL) - \alpha$$

Assumption

$$p_r \beta_r - p_s \beta_s = k$$

... difference in expected marginal return constant

INDIVIDUAL LENDING SWITCH LINE

Switch Line: Locus of contracts (r, L) along which the borrower is indifferent between risky and safe project

$$V_r > V_s$$

$$p_r(\beta_r L - rL) - \alpha > p_s(\beta_s L - rL)$$

$$L > \frac{\alpha}{\Delta p r + k} \quad (\text{Output threshold})$$

Northeast of the switch line: Sunk cost investment α is overwhelmed by increased expected marginal productivity of risky project k and saving on the expected interest rate payment $\Delta p r$.

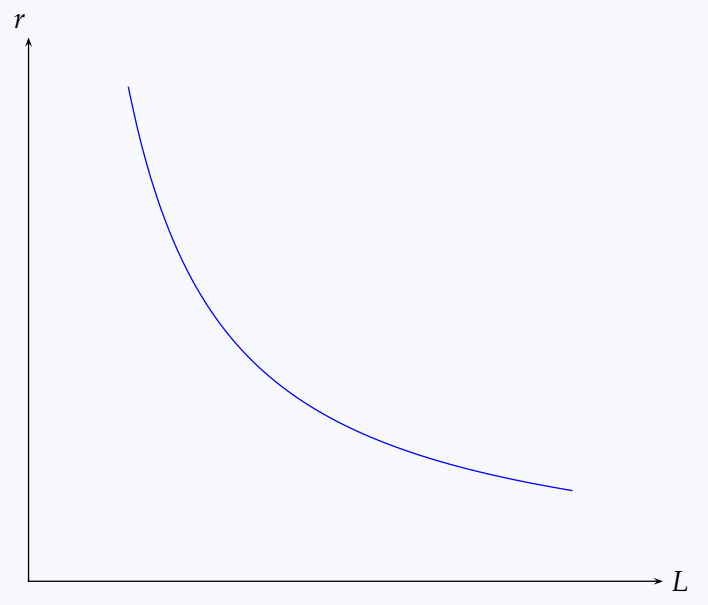


Figure: Switch Line

LENDER'S ZERO PROFIT CONDITION

Risk adjusted interest rate

$$r = \frac{\rho}{p_i} \quad i = s, f \quad (\text{L-ZPC})$$

Optimal Contract (r^*, L^*) : Switch line & (L-ZPC)

Maximum loan size & Interest Rate

$$L^* = \frac{\alpha}{\Delta p \left(\frac{\rho}{p_s} \right) + k}$$

$$r^* = \frac{\rho}{p_s}$$

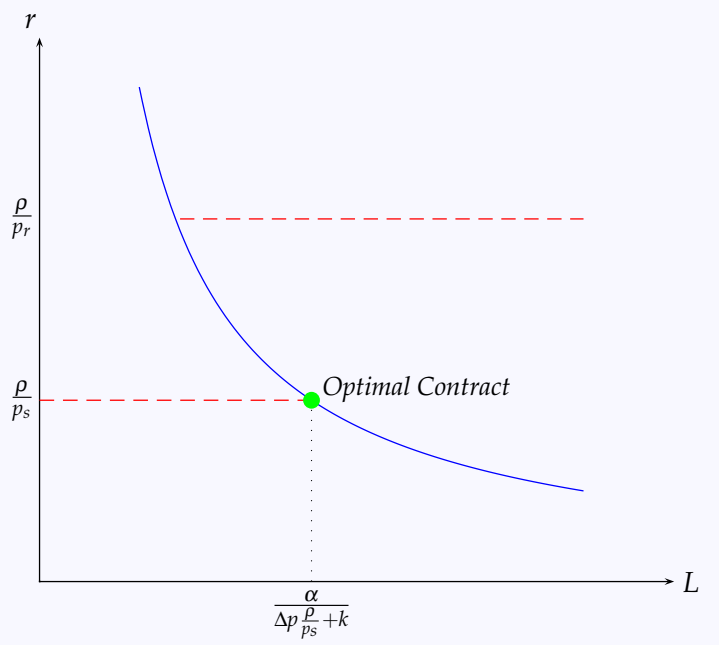


Figure: Switch Line and Optimal Contract under Individual Lending

GROUP LENDING

Borrower's payoffs

$$V_{ss} = p_s(\beta_s L - rL) - p_s(1 - p_s)cL$$

$$V_{rr} = p_r(\beta_r L - rL) - \alpha - p_r(1 - p_r)cL$$

Joint liability payment c incurred with probability $p_i(1 - p_i)$

- Payoffs ↓ due to the joint liability payment c
- Payoffs ↑ due to larger loans

GROUP LENDING SWITCH LINE

Group Lending Switch Line: Lender's Zero Profit Condition:

$$L = \frac{\alpha}{\Delta p r + k - \Delta p(p_s + p_r - 1)c}$$

$$r = \left(\frac{\rho}{p_s}\right) - \left(\frac{1 - p_s}{p_s}\right)c$$

Maximum Loan Size in Group Lending:

$$L^* = \frac{\alpha}{\Delta p \left(\frac{\rho}{p_s}\right) + k - \varphi c}$$

where $\varphi = \Delta p \left(\frac{1 - p_s}{p_s} + (p_s + p_r - 1)\right)$

- Joint liability payment lets borrowers get larger loans
- ... L^* is increasing in c

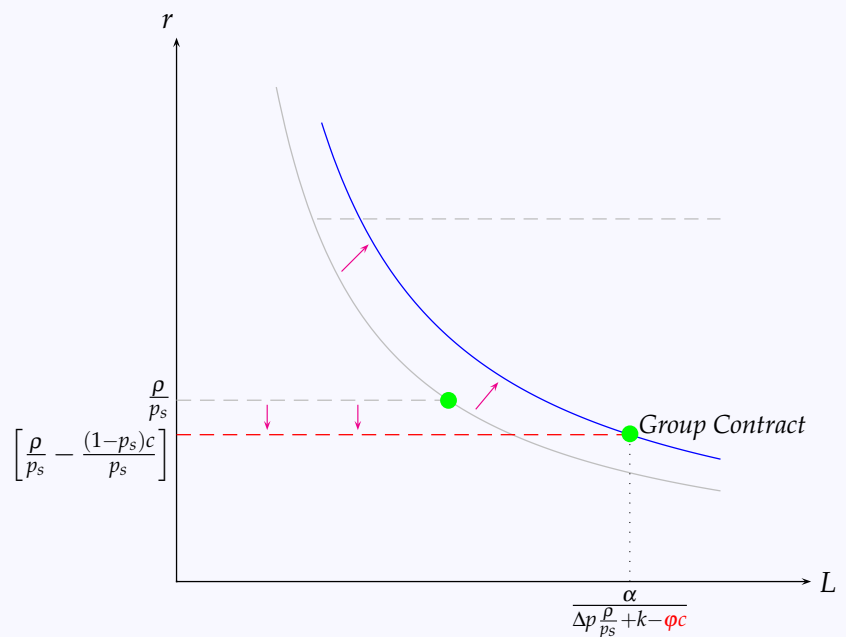


Figure: Switch Line and Optimal Contract under Group Lending

PROJECT CHOICE SUMMARY

Lender curtails loan size to prevent borrowers undertaking risky loans with significantly high sunk cost

Individual liability loans	Joint liability group loans
1. Borrower pay ρ	1. Borrower pay ρ
2. Lower risk exposure	2. Higher risk exposure
3. Small Loans	3. Larger Loans

May explain why we find the poorer section of our society are not able to undertake profitable investment

Borrowers interact cooperatively and not strategically amongst themselves	Can lender do better by making the borrowers interact strategically amongst themselves
-----------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------

LOAN CONTRACT & STRATEGIC DEFAULT

Lender offers borrower the following contract:

1. Loan amount 1
2. Interest rate r
3. Duration ... 1 time period

After output realisation, borrower chooses:

- Involuntary Default:* Insufficient output for repayment.
... borrower has no option but to default
- Strategic Default:* Sufficient output for Repayment obligations
... borrower chooses to default

Assume away *Involuntary Default* to focus on *Strategic Default*.

Output realisation is always greater than r

CONTRACT ENFORCEMENT

Interaction between the lender(s) and wealth-less borrower(s) in the context of credit markets.

Explore the interaction between between borrower's limited ability to enforce contracts and borrower's incentive to default strategically.

Ideal world: Lender has unlimited ability to enforce contracts, i.e., punish strategic defaulters → Obtains repayment with certainty.

Limited enforcement capability → lender obtains repayment in the cases where the punishment exceeds the borrower's benefit from defaulting.

ENFORCEMENT SET UP & PENALTY FUNCTION

Project: 1 unit of capital investment yields x . x is distributed on $[\underline{x}, \bar{x}]$ according to the distribution function $F[x]$.

Intuition: There is some external factor beyond the control of the borrower affecting the value of the project output.

If the borrower could affect the value, it would be a moral hazard environment.

PROJECT EXAMPLES

A buyer in the UK borrows and buys a flat in London
The value of the flat in the future depends on the housing market and is beyond the control of the buyer.

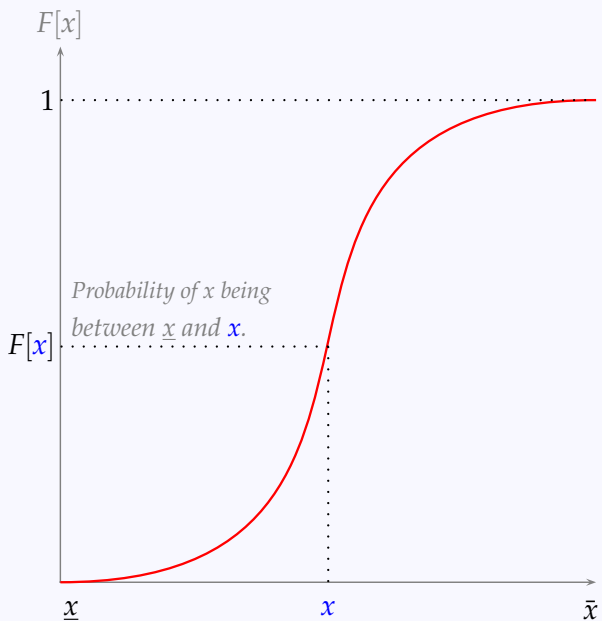
A farmer in Kenya borrows and buys a buffalo The output of the buffalo depends on the price of the milk in the local market which is beyond the farmer's control.

↪ **Lender's Penalty:** In case of a threat of default on the borrowing, the lender can penalise the borrower by confiscating the project output, i.e., the flat or the buffalo.

↪ The higher the value of the flat or milk, the more reluctant the borrower is to

... part with the project
... default on the loan

DISTRIBUTION OF x



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SET UP & PENALTY FUNCTION

Project: 1 unit of capital investment yields x . x is distributed on $[\underline{x}, \bar{x}]$ according to the distribution function $F[x]$.

Penalty Function $p(x)$: the output contingent penalty that the lender can impose on the borrower(s) once the project has been completed and the output x has been realised.

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PENALTY FUNCTION

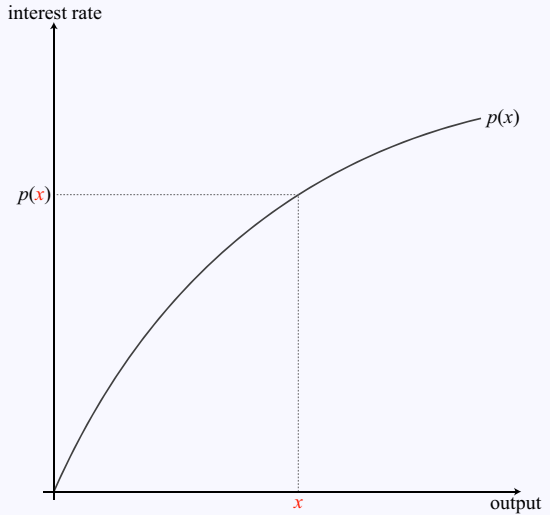


Figure: Penalty Function

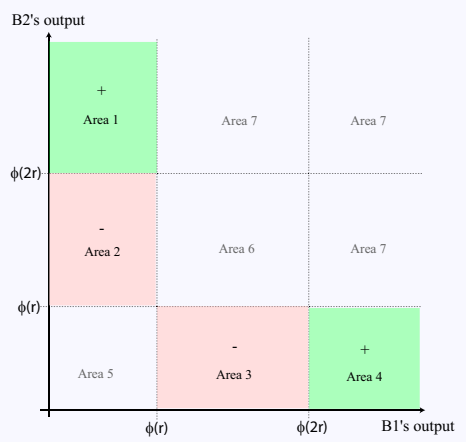
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THRESHOLD FUNCTION

Threshold Function $\phi(r)$: Given r , it gives the threshold output beyond which the borrower would choose to repay. Conversely, if the project output is below this threshold output, the borrower would choose to default strategically.

Inverse of the penalty function.

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- Under Area 1, B1 (B2) would have defaulted under individual lending. The loans are repaid under group lending.
- Under Area 2, B2 would have repaid under individual lending but does not pay under group lending due to joint liability.

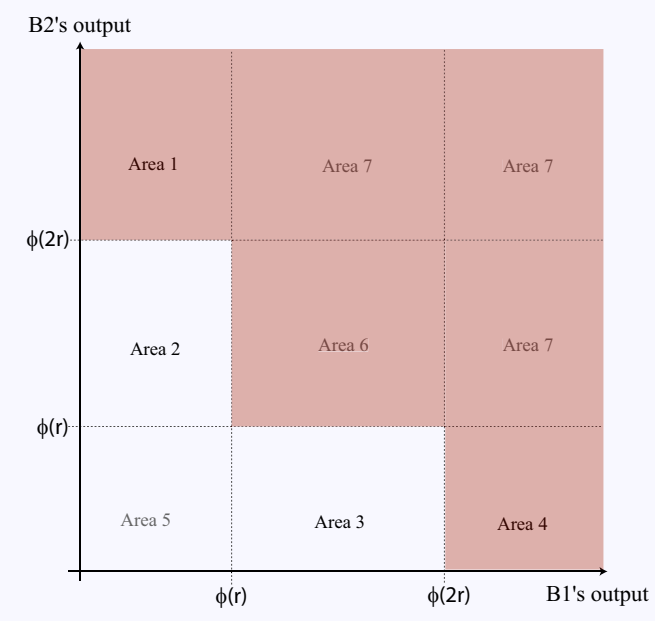


Figure: Repayment Area in Group Lending without Social Sanction

GROUP LENDING WITH SOCIAL SANCTION

Analyse the group member's ability to social sanction each other, which can be used to amplify the effect of lender's penalty.

Group members impose a negative externality on each other when one group member would like to pay off her own loan but defaults because her peer is going to default.

Social Sanction s: If a group member imposes a negative externality on her peer, she faces a social sanction s in response.

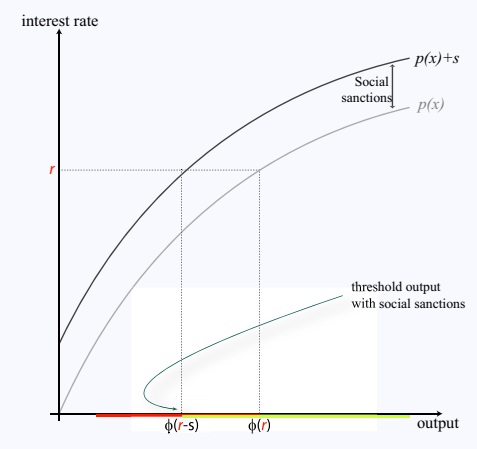


Figure: Threshold Output with Social Sanctions

