2010

Paper I

A1. $x = 1, \frac{3}{2}$ or 2. 2. (a) $-2xe^{-x^2}$; (b) $2\sinh x \cosh x$. 3. (a) $x\ln x - x$; (b) zero.

4. (a) 1. 5. Radius 3 centre (3,1). 6. $x = e^2, k = e^{-2}$. 7. (a) 1000; (b) $\frac{(n+1)(n+2)}{2}$. 8. –

9. $x = -2, x = -4$ (if one allows the negative root). 10 $x = \frac{\pi}{8}$ or $\frac{5\pi}{8}$.

B11. (a) 2.0305; (b)(i) $x + x^2 + \frac{x^3}{3}$; (ii) $x + \frac{1}{2}x^2 - \frac{2x^3}{3}$.

12. (a) $(1-p)^F$; (b) $\binom{F}{n}p^n(1-p)^{F-n}$; (c)$Fp, Fp(1-p)$; (d) $\binom{F-1}{n}p^n(1-p)^{F-n}$;

(e) $\sum_{k=0}^{F-n-1}\binom{n-1+k}{k}p^n(1-p)^k$.

13. (a) $1, 3 \pm \sqrt{15}$, eigenvector for $\lambda = 1$ is $(1\ 1\ 3)^T$ (lost the will to live beyond that);

(b) $\left(-\frac{3}{2}\ \frac{7}{2}\ -\frac{3}{2}\right)^T$.

14. (a)(i) $\frac{12}{13}, \frac{5}{13}$; (ii) $0, \frac{\pi}{6} + 2n\pi$; (iii) $0, -1$; (iv) $-\frac{1}{2}, -\frac{\sqrt{3}}{2}$; (b) $2^{\frac{1}{4}}e^{i\left(\frac{\pi}{12}+\frac{n\pi}{2}\right)}, n = 0, 1, 2, 3$.

15. (a) $\mathbf{x} = \frac{\mathrm{a}-(\mathrm{b}\wedge\mathrm{a})}{1+|\mathrm{b}|^2}$ (?); (b) $\mathbf{x} = \mathbf{a} + \left(\frac{\mathbf{a\cdot b}}{\mathbf{1-b\cdot c}}\right)\mathbf{c}$, if $\mathbf{b}\cdot\mathbf{c} = 1$ then $\mathbf{x} = \mathbf{a} + \gamma\mathbf{c}$ for any γ.

16. -.

17. (a) $y = -\frac{1}{x}\ln(1 + e^{-1} - x)$; (b) $y = \left(4(1+x^3)\right)^{\frac{1}{3}}$;

(c) $y = Ae^x + B, y = 4e^{x-1} - \left(\frac{x^2}{2} + x + \frac{3}{2}\right)$.

18. (a)(i) $at^{a-1}y + t^{2a-1}b^a axy'$; (ii) $t^{4a}b^{3a}y'''$; (b) $a = -\frac{1}{3}, b = 3$.

19. (b) $2e\delta x + e\delta y$; (c) $u = A\sin 3x \sin 4y + B\sin 4x \sin 3y$.

20. (a) (i) Yes (e^2); (ii) No; (b) $2 < x < 4$; (c) (i) 4; (ii) $\frac{1}{9}$.

Paper II

A1. $\theta = 0.906$ rad. 2. 2 3. $(\pm 1, 0)$. 4. $\pm\frac{\sqrt{28}}{3}$. 5. $-\frac{5}{41}, \frac{37}{41}$. 6. $\frac{1}{2} - \frac{\sqrt{3}\left(x-\frac{\pi}{3}\right)}{2} - \frac{\left(x-\frac{\pi}{3}\right)^2}{4}$. 7. $y = -2x + 1$.

8. $\frac{11}{36}$ 9. $y = 1$. 10. $y^2z^2 + 4y^3 + x^3y^2$.

B11. (a) $x = r\cos\theta, y = r\sin\theta$; (c) AC^{-2}, C^{-1}.

12. (b) $x = X_0\cos\sqrt{\frac{k}{m}}t, y = V_0\sqrt{\frac{m}{k}}\sin\sqrt{\frac{k}{m}}t$.

13. (a) minimum at (0,0); (b) Saddles at $(\pm 4,0)$, Minimum at (0,2), Maximum at $(0,-2)$.

14. (a) (i) 1.

15. $\frac{\sin\mu\pi}{\mu\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{2\mu(-1)^n\sin\mu\pi}{\mu^2-n^2}\cos nx$; $A = 3, B = 3$.

16. $(0, 2, 2), (0, 0, 0)$; F_1: (a) 1, (b) $\frac{\pi}{2}$; F_2: (a) 0, (b) 0.

17. (a) (i) $\frac{16}{35}$; (ii) $\frac{7}{16}$; (b) $\frac{\binom{34}{3}\binom{64}{4}}{\binom{100}{7}}$; (c) (i) $\alpha = P(B|A)P(A), \beta = (P(B|A) - 1)P(A) + 1$;

(ii) $\alpha = \mathrm{Min}\left(P(B|A), P(B|\bar{A})\right), \beta = \mathrm{Max}\left(P(B|A), P(B|\bar{A})\right)$.

18. (a) 224; (b) $\frac{1}{2}(e-1)$; (c) $\frac{81\pi}{4}$

19. (a) $C = 6$; (b) $D = -1, E = 5$.

20. (a) $H_0 = 1, H_1 = 2x, H_2 = 4x^2 - 2$.

2011

Paper I

A1. 1. 2. (a) $-\frac{1}{2}\ln(3-2x)+c$. 3. (a) $(0,1)$; (b) $(e,e^{1/e})$.

4. (a) spiral, starting from (0,4π) and going clockwise into origin; (b) $7\pi/2$. 5. $\pi{:}\,3\sqrt{3}$.

6. $\pm\frac{\pi}{6},\pm\frac{5\pi}{6}$. 7. $\ln\frac{1}{4}$ or $\ln\frac{3}{4}$. 8. $e^{-x^2}\sin x$. 9. $\frac{1}{x\ln x}$. 10. (a) 10^6; (b) $\frac{8}{7}$.

B11. (b)(i) $\frac{1}{3}-\frac{x^2}{54}$; (ii) $3\ln 2+\frac{3x}{2}-\frac{3x^2}{8}+\frac{x^3}{8}$; (iii) $1+x+\frac{1}{2}x^2$.

12. (a) $\frac{1}{\bar{s}}e^{-\frac{s}{\bar{s}}}$; (d) $\frac{\bar{t}}{\bar{s}+\bar{t}}$; (f) $\left(1-\frac{\bar{t}}{\bar{s}+\bar{t}}\right)e^{-\frac{r_0}{\bar{s}}}$; (h) $\bar{s}$.

13. (a) (i) 16; (ii) $\frac{1}{16}\begin{pmatrix}5 & 3\\ 3 & 5\end{pmatrix}$; (iii) 2 and 8; $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$; (b) $b=5,c=-6,d=5$

(c) $2x'^2+8y'^2=1, B=\begin{pmatrix}1 & 1\\ 2 & -2\end{pmatrix}$.

14. (d) $a_m=b_m=0\ if\ m\neq 1, a_1=-\sqrt{3}, b_1=1$; (e) $2\sin(x-\frac{\pi}{3})$; (g) 4π.

15. (d) $s=\frac{[\mathrm{a,b,n}]}{[\mathrm{b,m,n}]}\ t=\frac{[\mathrm{a,b,m}]}{[\mathrm{a,m,n}]}$.

16. (a) $y=2\ln x-1$; (b) $y=(x^2-1)e^{-x^2}$; (c) $y=\frac{8}{3}\cos 2x-\frac{\sqrt{2}}{3}\sin 2x+\frac{1}{3}\cos x+\frac{1}{3}\sin x$.

17. (b) (i) $2,-\frac{\pi}{3}$; (ii) $\sqrt{5},\tan^{-1}2$; (iii) $\sqrt{2},\frac{5\pi}{12}$; (b) $e^{\pm i\pi(\frac{2}{9}+\frac{2n}{3})}$.

18. (b) first not exact, second is exact (c) $\mu=1/\sin x, y=-\frac{1}{x}\ln(\sin x)+\frac{c}{x}$; (d) $\left(\frac{\partial a}{\partial d}\right)_e$.

19. (a) zero; (b) π (or $-\pi$, if you take **dS** to be in negative y direction); (c) $4\pi k r^3$; (d) $\pm 4\pi$.

20. (a) All cases continuous and differentiable except: $x=0$ and $\pm\pi$, f_0 is continuous and differentiable, f_1 and f_2 are not even continuous (limit is non-zero but function is defined to be zero there). At $\pm\frac{\pi}{2}$, I think the question is just a more tiresome version of the supervision sheet question Mich Term Examples I J1. At $\pm\frac{\pi}{2}$, f_0 is not even continuous; f_1 is continuous but not differentiable; f_2 is continuous and differentiable. The idea of plotting f_1 and/or f_2 is horrendous. (b) $x^2+\frac{1}{6x^8}+O(\frac{1}{x^{18}})$.

Paper II

A1. $c=6$. 2. $\ln(x^2+3x-2)+c$. 3. $x=-1,-5$, discontinuities at $x=-2,1$. 4. -.

5. $Re=\frac{14}{25}, Im=-\frac{23}{25}$. 6. $1+\frac{(x-1)}{2}-\frac{(x-1)^2}{8}$. 7. $y=-\ln(1-\frac{1}{2}\ln(x^2+1))$.

8. Circle, radius 2, centre (0,1). 9. $y=12x-15$. 10. Zero.

B11. (a) (i) π; (ii) Zero; (iii) Zero; (b) (i) and (ii) are both $\pm 10e\pi\mathbf{k}$ (don't know which until the rim traversal direction is specified?)

12. -.

13. (b) $-\sqrt{3}$.

14. (a) (i) 45; (iii) $\begin{pmatrix}1\\2\\-1\end{pmatrix}$, anything satisfying $x+2y-3z=0$, eg $\begin{pmatrix}1\\1\\1\end{pmatrix}$.

15. (a) $x=\frac{\mathrm{V_{0x}}}{\gamma}(1-e^{-\gamma t}), z=\left(\frac{\mathrm{V_{0z}}}{\gamma}+\frac{g}{\gamma^2}\right)(1-e^{-\gamma t})-\frac{gt}{\gamma}$;

(b) $z=\frac{g}{\gamma^2}\left[\ln\left(1-\frac{\gamma x}{\mathrm{V_{0x}}}\right)+\left(\frac{\gamma^2\mathrm{V_{0z}}}{\mathrm{g}}+\gamma\right)\frac{x}{\mathrm{V_{0x}}}\right]$.

16. (a) $E(\frac{2}{3},\frac{\pi}{6})$; (c) $\frac{\pi}{2a}(3a^2+b^2)$.

17. (a) $\frac{(32!)^3}{64!(16!)^2}$; (b) $l=\frac{3}{4}, p=\frac{4}{5}$.

18. (a) $\frac{R^3}{6}$; (b) $\frac{4}{3}$; (c) 8π.

19. (c) $\Psi(x, y) = \frac{2V}{\pi}\tan^{-1}\left(\frac{2y}{1-x^2-y^2}\right)$.

20. (a) $L_{10} = \frac{1}{10!}x^{10} - \frac{10}{9!}x^9 + \cdots$.

2012

Paper I

A1. $f = -1\ x = 2$. 2. $y = -\frac{1}{4}x + \frac{13}{4}$. 3. $p = 7; x^2 + 2x + 4$ rem 5.

4. (a) $(1-2x)e^{-2x}$; (b) $\frac{1}{x^2} - \frac{\ln x}{x^2}$. 5. $\pi^2 - 4$. 6. $\frac{1}{x^2-1} + \frac{2}{x-1} + \frac{2}{x+2}$. 7. $y = 3e^{x^3-1}$.

8. (a) $\frac{1}{8}$; (b) $84x^3$. 9. - 10. $\frac{\pi^2}{2}$.

B11. (a) $a = 0$ or $b = \pm c$; (b) $k(6 \quad 1 \quad -3)^T$; (c) $a = 0\ b = -\frac{7}{3}c = \frac{5}{3}$ (d) no.

12. (a) (i) $\mathrm{Re} = -2\ \mathrm{Im} = \frac{3}{2}$; (ii) $\mathrm{Re} = 0\ \mathrm{Im} = 32$; (iii) $\mathrm{Re} = \frac{5}{4}\ \mathrm{Im} = 0$; (b) $\sqrt[4]{2}e^{(\frac{3\pi}{8}+n\pi)i}$;

(c) $\frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$; (d) $z = (1+2n)i\pi$.

13. (a) $\mathrm{y} = -\frac{\mathrm{x}}{\ln \mathrm{x} + \mathrm{c}}$ or $\mathrm{y} = 0$; (b) $y = \frac{1}{x}\left(k - \frac{1}{2}x^2\right)$; (c) $y = \frac{a(k\sin mx - m\cos mx) + (k^2+m^2+am)e^{-kx}}{k^2+m^2}$.

14. (a) $(\mathbf{a.c})\mathbf{b} - (\mathbf{a.b})\mathbf{c}$; (b) lines parallel; (c) $\mathbf{a.c} = \mu, \mathbf{a.b} = \lambda$; (e) $\sqrt{\lambda^2/|\mathbf{b}|^2 + \mu^2/|\mathbf{c}|^2}$.

15. (a) $\left(\frac{3}{4}, \frac{3\sqrt{3}}{4}\right)$; (b) $\frac{3\pi}{2}$; (c) 8.

16. (a) $-\frac{2}{5}$; (b) $(0, \pm\sqrt{2}), (\pm\sqrt{2}, 0)$; (c) saddle point.

17. (a) 0 unless $m = n \neq 0$ in which case L, or $m = n = 0$ in which case $2L$;

(b) $\frac{e-e^{-1}}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n(e-e^{-1})}{1+n^2\pi^2}(\cos n\pi x - n\pi \sin n\pi x)$.

18. (a) (i) $\left(\frac{3}{4}\right)^{24}$; (ii) $\left(\frac{1}{4}\right)^{24}$; (iii) 18; (iv) $\frac{3}{2}\sqrt{2}$; (b) (i) $\int_\alpha^\beta f(x)dx$; (ii) $\int_\alpha^\beta xf(x)dx$, $\int_\alpha^\beta (x-\mu)^2 f(x)dx$;

(c) (i) $A = \lambda^2$; (ii) $2/\lambda$, $2/\lambda^2$.

19. (a) (ii) continuous and differentiable; (iii) zero; (iv) $\frac{(-1)^n}{n!}$; (b) (i) 2; (ii) limiting value = 2 always.

20. (a) $h = 1\ w = l = 2$; (b) $\left(\sqrt[4]{\frac{1}{2}}, \sqrt[4]{2} - \sqrt[4]{\frac{1}{2}}\right)$.

Paper II

A1. $a = 2\ b = 1\ c = 3$. 2. $\frac{1}{2}(e-1)$. 3. $\frac{1}{5}\begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$. 4. Re=$-\frac{7}{41}$, Im = $\frac{22}{41}$.

5. $\ln 4 + (x-2) - \frac{1}{4}(x-2)^2$. 6. $\omega = \frac{2}{3}$. 7. $\frac{2}{3}$. 8. $3x + 6z, (0, 2x, 3y)$. 9. $\frac{1}{2}$.

10. $x = 0, \pm\sqrt{2 - \pi/2}, \pm\sqrt{2 + \pi/2}$.

B11. (c) $\frac{1}{\sqrt{2}}(1 \quad 0 \quad 1)^T, (0 \quad 1 \quad 0)^T, \frac{1}{\sqrt{2}}(1 \quad 0 \quad -1)^T$ (f) $\mathbf{B}^{-1} = \mathbf{B}^T$

12. (a) $\cos^{\mathrm{k}} x(1 - \sin^3 x)^n, -\cos^{\mathrm{k}} y(1 - \sin^3 y)^n, 0$; (b) $f(n) = \frac{3n}{3n+1}$; (c) $\frac{81}{140}$; (d) $\frac{1}{3(n+1)}$.

13. (a) $y = \frac{1}{3}e^x + \frac{2}{3}e^{-2x}$; (b) $y = (A + Bx)e^{-3x} + \frac{x}{3} - \frac{1}{9}$; (c) $y = Ae^{-x} + Be^{-2x} - \frac{3}{4} - \frac{x}{2} + \frac{1}{20}e^{3x}$.

14. (a) (i) $\sqrt{\frac{2}{3}}$; (ii) $A = \frac{\sqrt{6}}{2}, \hat{\boldsymbol{n}} = \frac{1}{\sqrt{6}}(1 \quad 1 \quad 2)^T, -\frac{1}{2}(1 \quad 1 \quad 2)^T$; (iii) $\frac{3}{\sqrt{6}}$; (b) 3.

15. (a) $V = \frac{kr^2}{2a^3} + A, V = -\frac{k}{r} + B$; (c) $B = 0\ A = -\frac{3k}{2a}$

16. (a) $162(t\cos^4 t + \cos^3 t \sin t - 3t\cos^2 t \sin^2 t) + 9(\cos^2 t - \sin^2 t) + 2, -7$; (b) $a = 2$; (c) $\frac{x^3}{3} + \frac{x^2 y}{2} - xy^2 = k$.

17. (b) $\frac{23}{16}$; (c) $\ln 2 + \frac{x}{2} + \frac{x^2}{8}$.

18. (a) $\frac{13}{18}, \frac{5}{9}, \frac{4}{5}, \frac{8}{13}, \frac{4}{9}$; (c) $\frac{1}{6}$; (d) $\frac{8}{17}$.

19. (a) $f(a,a) + \int_0^a \frac{\partial f(x,a)}{\partial a}dx; -\frac{1}{a^2} + 2e^{-a^2} + \frac{e^{-a^2}}{a^2}$; (b) $\frac{1}{b+1}$.

20. (c) $u(x,t) = ax$.

2013

Paper I

A1. (b) $\max y = \sqrt{2}$. 2. (a) $(\frac{1}{2}, \frac{1}{4})$; (b) $\pi/2$. 3. (a) $y = \ln(e^x + c)$; (b) $y = \ln(e^x + 1)$.

4. $-\frac{1}{6}, 8.5$. 5. (b) 3. 6. (a) $\frac{x^{m+1}}{m+1}\left(\ln x - \frac{1}{m+1}\right) + c$; (b) $\frac{(\ln x)^2}{2} + c$. 7. (a) $\frac{5}{x-3} + \frac{2}{x+7}$; (b) $(x-3)^2 + 5$.

8. (a) $a^x \ln a$; (b) $-\cos(\cos x)\sin x$. 9. Assuming $b(0) = 0$: (a) $\sqrt{100 + 16t^2}$; (b) $12/\sqrt{10}$. 10. $\frac{44}{15}$.

B11. (a) $(1\ 2\ 3)^T$; (b) *a* and *b* axes plus circles radii 1,2,3... centred on origin.

12. (a) $2, -1 \pm i\sqrt{3}$; (b) $2 + i$ or 1; (c) -2,$-i$; (d) $\frac{3}{t^2+1}$; $\frac{6}{(t^2+1)^{3/2}}$.

13. (a) $y = (1 + x^2)^{-2}(\tan^{-1} x + 1)$; (b) $x = y \ln y - 2 + cy$; (c) $y = \frac{1}{x^2(\sin x + c)}$.

14. (c) $x - 4y - z + 5 = 0$; (e) $75\sqrt{2}/8$.

15. (a) $az = r^2$; (b) $\frac{a^2\pi}{6}\left(\left(\frac{4h}{a} + 1\right)^{\frac{3}{2}} - 1\right)$; (c) (i) $\pi a h^2/2$; (ii) $(0,0,2h/3)$.

16. (a) Saddles at $(0,0), (\pm 1,0), (0,\pm 1)$, minima at $\pm\left(\frac{1}{2}, \frac{1}{2}\right)$, maxima at $\pm\left(\frac{1}{2}, -\frac{1}{2}\right)$;
(b) (ii) $y = \sin 2x \cos 5t$.

17. (a) $L\left(\frac{a_0^2}{2} + \sum a_n^2 + b_n^2\right)$; (b) $\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{2}{(2n-1)^2\pi}\cos(2n-1)x - \frac{\sin nx}{n}$

18. (a) (i) $\frac{100}{231}$; (b) (i) $\frac{2}{5}$; (ii) $1 - 0.5\alpha$; (iii) $\frac{0.8 - 0.2\alpha}{2 - \alpha}$.

19. (b) $h > f > g$; (d) diverges if $p \leq 1$ converges if $p > 1$.

20. (a) $\frac{4a^3}{3\sqrt{3}}$.

Paper II

A1. (a) $(-4 \quad -9 \quad 13)^T$; (b) -4. 2. (a) 1; (b) $\frac{u}{\sqrt{1-u^2}}$. 3. $-\frac{5}{13}, -\frac{12}{13}$. 4. $\frac{7}{\sqrt{5}}$. 5. 0: $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$, 5: $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

6. $\frac{1}{2} - \left(x - \frac{\pi}{4}\right)^2$. 7. (a) $r = \sqrt{3}\ \phi = \frac{\pi}{3}\ \theta = \frac{\pi}{6}$. 8. (a) $(\cos xy + xy\cos^2 xy - xy \sin xy)e^{\sin xy}$;
(b) $2xz + 3y^2 z$. 9. (a) $-\frac{\boldsymbol{r}}{r^3}$. 10. (a) $\frac{1}{216}$ (b) $\frac{91}{216}$.

B11. (b) $\frac{\alpha}{\alpha - 1}$, $\alpha > 1$; (c) $\frac{\alpha}{(\alpha-2)(\alpha-1)^2}$, $\alpha > 2$; (d) $F(x) = 1 - x^{-\alpha}$, $x = 2^{1/\alpha}$; (e) $\frac{1-3^{-\alpha}}{1-6^{-\alpha}}$.

12. (a) (i) $(\boldsymbol{a}.\boldsymbol{b})^2$; (ii) $\boldsymbol{a}.\boldsymbol{b} + |\boldsymbol{a}|^2|\boldsymbol{b}|^2$; (iii) $(\boldsymbol{a}.\boldsymbol{a})^6$; (b) (i) zero; (ii) $\begin{pmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}$; (iii) $\begin{pmatrix} -\frac{1}{4} & 0 \\ 0 & -\frac{1}{4} \end{pmatrix}$,
$\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$; (iv) $\begin{pmatrix} 0 & -\frac{1}{8} \\ \frac{1}{8} & 0 \end{pmatrix}$; (v) $\frac{(-1)^n}{2^{2n}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\frac{(-1)^n}{2^{2n+1}}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

13. (a) $\sqrt{\pi}$; (b)(i) $I_n = \frac{n-1}{2} I_{n-2}$; (ii) $I_1 = 0, I_2 = \frac{1}{2}\sqrt{\pi}$ (c)(ii) zero; (d) $\frac{\pi}{3\sqrt{3}}$.

14. (a) $y = -\frac{1}{3}x^3 - \frac{1}{3}x^2 - \frac{8}{9}x + d + ce^{3x}$; (b) (i) $y = A\cos 2x + B \sin 2x + \frac{1}{3}\sin x$;
(ii) $y = A\cos 2x + B\sin 2x - \frac{x}{4}\cos 2x$; (iii) $y = A\cos 2x + B\sin 2x + \frac{1}{3}\sin x - \frac{x}{4}\cos 2x$;
(c) $y = 3e^t - e^{2t}$.

15. (a) $(a, 0)$; (b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; (c) $y \to \pm\frac{bx}{a}, x \geq a$; (d) difference in distances is $2a$.

16. (a) (i) $(6x^2 \quad 2y - 1 \quad 6z)^T$; (ii) $\frac{1}{\sqrt{2}}(1 \quad 1 \quad 0)^T$; (iii) $\frac{27}{\sqrt{2}}$; (iv) $\int_0^{2\pi} V d\theta = 62\pi$.

17. (a) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$, $\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$, $\left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial T}{\partial P}\right)_S$; (b) $U = cT - \frac{a}{V}$.

18. (b) $(0,0)$; (c) $x \geq 0$, $y = x^2 \pm x^{5/2}$; (d) Min and zero on both branches at $x = 0$, max on negative branch at $x = 16/25$, zero on negative branch at $x = 1$.
19. (b) $-a^2\pi$.
20. (a) $f(b(\theta),\theta)\frac{db}{d\theta} - f(a(\theta),\theta)\frac{da}{d\theta} + \int_{a(\theta)}^{b(\theta)} \frac{\partial f}{\partial \theta} dx$; (c) $F(t) = \sqrt{\frac{\pi}{2}} e^{-t^2/2}$.

2014

Paper I

A1. (a) $-\frac{2x}{(x^2+4)^2}$; (b) $\cos x\, e^{\sin x}$. 2. (a) $-a^{-x}\ln a$; (b) $\ln x \ln(\ln x) - \ln x + c$. 3. (a) $\frac{1}{2}\ln 2$; (b) $-\ln 2$.

4. (a) $\tan y = c - \cos x$; (b) $y = 3e^{3x}$. 5. (a) $\left(\frac{4}{3}, \frac{7}{3}\right)$ and $(-1,0)$; (b) ellipse centre $(1,0)$.

6. (a) $1/40\pi$; (b) $1/3\sqrt{2}$. 7. -. 8. (a) $x = \frac{7}{3}$ or -1; (b) Min. 9. $\frac{2}{3}$. 10. $y = -\frac{1}{2}x - \frac{3}{4}$; $\frac{5}{x-4} + \frac{8}{x+9}$.

B11. (a) Det A=0, Tr A=6, at least one eigenvalue zero; (b) $\lambda = 0\ \mathbf{x} = (1 \quad 2 \quad 1)^T$,
$\lambda = -6\ \mathbf{x} = (1 \quad 0 \quad -1)^T$, $\lambda = 12\ \mathbf{x} = (1 \quad -1 \quad 1)^T$; *(c)* $\mathbf{e} = k(1 \quad 2 \quad 1)^T$; $\mathbf{Ar}$ lies in the plane $\perp (1 \quad 2 \quad 1)^T$.

12. (a) modulus = $\sqrt[6]{2}$, arg = $\frac{\pi}{4} + \frac{2n\pi}{3}$; (b) $z = i\left(-\frac{\pi}{4} + n\pi\right)$; (c) 1, $(1+i)$.

13. (a)(i) $y = \frac{4}{3}e^{-3x} + \frac{8}{3}$; (ii) $y = e^{\sin x} - (1 + \sin x)$; (b) $(2+2x)e^{-3x} - e^{-4x}$.

14. (a) $(\mathbf{a} - \mathbf{c}).(\hat{\mathbf{b}} \wedge \hat{\mathbf{d}})/|(\hat{\mathbf{b}} \wedge \hat{\mathbf{d}})|$; (b) Quick: just say it's a plane, slower $\mathbf{p.q} \neq 0$: $\lambda\mathbf{p} + (k - \lambda\mathbf{p.p})/\mathbf{p.q} + \nu\,\mathbf{p} \wedge \mathbf{q}$, $\mathbf{p.q} = 0$: $k\mathbf{p}/\mathbf{p.p} + \mu\mathbf{q} + \nu\,\mathbf{p} \wedge \mathbf{q}$; (c) $r = 1, \phi = t, \theta = \cot^{-1} t$.

15. (a)(ii) $\frac{9\sqrt{3}}{8} - \frac{\pi}{4}$; (iii) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$; (b) $x_0\left(a\sqrt{\pi}\right)^3$.

16. (a) Saddles at $\pm(3{,}4)$, minimum at $(5{,}0)$, maximum at $(-5{,}0)$;
(b) Saddle at $(0{,}0)$, minima at $\pm(3{,}3)$.

17. (b) $\cosh \pi$.

18. (c) (i) $\frac{28}{45}$; (ii) $\frac{16}{45}$; (iii) $\frac{1}{45}$; mean=$\frac{2}{5}$; var = $\frac{64}{225}$.

19. (a) $|x| < 2$, diverges at $x = -2$, converges (conditionally) at $x = 2$; (b) $1 - \frac{3x}{2} + \frac{11}{8}x^2$;
(c)(i) diverges; (ii) converges; (d) $S(x) = (x^3 - 2x^2 + 2x)e^x - 2x$.

20. (a) $\sqrt{8}$.

Paper II

A1. (a) $a = 1, b = \frac{4}{3}$. 2. (a) Re=$\cosh\alpha\cos\beta$, Im=$\sinh\alpha\sin\beta$. 3. $x + \frac{x^2}{2}$. 4. $y = Ae^x + Be^{3x}$,
$y = x + \frac{4}{3}$ 5. div = $-e^{-x}\cos z - e^{-y}\sin z$, curl = $(-e^{-y}\cos z \quad -e^{-x}\sin z \quad 0)^T$.

6. $u = \cos(x - ct)$. 7. e^{ϕ} and $e^{-\phi}$. 8. $\left(\pi, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right), \left(\pi, \frac{3\pi}{2}\right), \left(\frac{3\pi}{2}, \pi\right)$. 9. (a) zero; (b) $-\frac{2}{3}$.

10. (a) 1 (b) $2^{-\alpha} - 3^{-\alpha}$.

B11. (a) $\frac{p_0}{p_0+p_1}$; (b) $A = \sigma^{-2}$, mean = $\sigma\sqrt{\frac{\pi}{2}}$; (c) $\binom{m+n}{m}\theta^m(1-\theta)^n$.

12. (a) (ii) $\mathbf{M}^{-1} = \frac{1}{8}\begin{pmatrix} 3 & 2 & 3 \\ 1 & -2 & 1 \\ -4 & 0 & 4 \end{pmatrix}$, no; (b) (ii) $x = -2, y = 3, z = 2$; (iii) $a = 1$ or $a = 3$
(iv) $z = 1, x + y = 2$.

13. (a)(i) $x + e^x + c$; (ii) $\frac{1}{6}\ln\left(x^2 + \frac{2x}{3} + \frac{2}{3}\right) - \frac{4}{3\sqrt{5}}\tan^{-1}\frac{3x+1}{\sqrt{5}} + c$; (b) zero; (c) $f(x)$;
(d) $2^N \sin(x^2 + x^6)$.

14. (a) $f = 2x^2 + \frac{x^2y^2}{2} + \frac{y^2}{2} + c$, $y = \sqrt{\frac{12-4x^2}{1+x^2}}$; (b) $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$, $\ln\psi = \int \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy$;
(c) $x^3y^2 + x^2y = k$ and hence $y = \frac{-x^2 \pm \sqrt{x^4 + 4kx^3}}{2x^3}$.

15. (b) $u_n = (1)(-2)(-5)\cdots(4-3n)x^n/3^n n!$; (c) $f(x) = \frac{1}{\sqrt{2}} - \frac{\sqrt{2}x}{\pi} + \frac{\sqrt{2}}{\pi^3}(4-\pi)x^2 + \cdots$;
(d) $\ln x^2 + \frac{1}{x} + \frac{1}{2x^2}$.

16. (a)(i) 0; (ii) 0; (b) only $\mathbf{F}_1$ is conservative, $\phi = x^3y^2z + c$, 2.

17. (a) $dp = \left(\frac{RT}{V} - \frac{2na}{V^2}\right) dn + \left(\frac{2n^2a}{V^3} - \frac{nRT}{V^2}\right) dV + \frac{nR}{V} dT$; (b) $\hat{\mathbf{r}}g'(r)$; (c)(i) 1; (ii) $\frac{1}{\sqrt{14}}(2 \quad -3 \quad 1)^T$.

18. (a) Maxima at $x = n\pi/2$ n odd, minima at $x = n\pi/2$ n even;

(c) $\frac{1}{2y\sqrt{\ln y}}, \frac{-1}{4y^2}(\ln y)^{-\frac{3}{2}}(1 + 2\ln y)$; (d) $\cos x$; (e) $-\sin x$; (f) $-\sin x$.

19. $c^2 = \frac{T}{\rho}$, $y = \frac{2}{c}\cos\frac{ct}{2}\sin\frac{x}{2} + \frac{2}{5c}\cos\frac{5ct}{2}\sin\frac{5x}{2}$.

20. (b) $\frac{1}{x^2}\cos x^3 + 3x\sin x^3 - \frac{1}{x^2}\cos 3x^2 - 6\sin 3x^2$; (c) $I'(a) = \frac{1}{1+a^2}$, $I(a) = \tan^{-1} a$, $I(1) = \frac{\pi}{4}$.