

2010

Paper I

- A1. $x = 1, \frac{3}{2}$ or 2. 2. (a) $-2xe^{-x^2}$; (b) $2 \sinh x \cosh x$. 3. (a) $x \ln x - x$; (b) zero.
 4. (a) 1. 5. Radius 3 centre (3,1). 6. $x = e^2, k = e^{-2}$. 7. (a) 1000; (b) $\frac{(n+1)(n+2)}{2}$. 8. -
 9. $x = -2, x = -4$ (if one allows the negative root). 10 $x = \frac{\pi}{8}$ or $\frac{5\pi}{8}$.
 B11. (a) 2.0305; (b)(i) $x + x^2 + \frac{x^3}{3}$; (ii) $x + \frac{1}{2}x^2 - \frac{2x^3}{3}$.
 12. (a) $(1-p)^F$; (b) $\binom{F}{n} p^n (1-p)^{F-n}$; (c) $Fp, Fp(1-p)$; (d) $\binom{F-1}{n} p^n (1-p)^{F-n}$;
 (e) $\sum_{k=0}^{F-n-1} \binom{n-1+k}{k} p^n (1-p)^k$.
 13. (a) $1, 3 \pm \sqrt{15}$, eigenvector for $\lambda = 1$ is $(1 \ 1 \ 3)^T$ (lost the will to live beyond that);
 (b) $\left(-\frac{3}{2} \ \frac{7}{2} \ -\frac{3}{2}\right)^T$.
 14. (a)(i) $\frac{12}{13}, \frac{5}{13}$; (ii) $0, \frac{\pi}{6} + 2n\pi$; (iii) $0, -1$; (iv) $-\frac{1}{2}, -\frac{\sqrt{3}}{2}$; (b) $2^{\frac{1}{4}} e^{i(\frac{\pi}{12} + \frac{n\pi}{2})}, n = 0, 1, 2, 3$.
 15. (a) $\mathbf{x} = \frac{\mathbf{a} - (\mathbf{b} \wedge \mathbf{a})}{1 + |\mathbf{b}|^2}$ (?); (b) $\mathbf{x} = \mathbf{a} + \left(\frac{\mathbf{a} \cdot \mathbf{b}}{1 - \mathbf{b} \cdot \mathbf{c}}\right) \mathbf{c}$, if $\mathbf{b} \cdot \mathbf{c} = 1$ then $\mathbf{x} = \mathbf{a} + \gamma \mathbf{c}$ for any γ .
 16. -.
 17. (a) $y = -\frac{1}{x} \ln(1 + e^{-1} - x)$; (b) $y = (4(1 + x^3))^{\frac{1}{3}}$;
 (c) $y = Ae^x + B, y = 4e^{x-1} - \left(\frac{x^2}{2} + x + \frac{3}{2}\right)$.
 18. (a)(i) $at^{a-1}y + t^{2a-1}b^aaxy'$; (ii) $t^{4a}b^{3a}y'''$; (b) $a = -\frac{1}{3}, b = 3$.
 19. (b) $2e\delta x + e\delta y$; (c) $u = A \sin 3x \sin 4y + B \sin 4x \sin 3y$.
 20. (a) (i) Yes (e^2); (ii) No; (b) $2 < x < 4$; (c) (i) 4; (ii) $\frac{1}{9}$.

Paper II

- A1. $\theta = 0.906$ rad. 2. 2 3. $(\pm 1, 0)$. 4. $\pm \frac{\sqrt{28}}{3}$. 5. $-\frac{5}{41}, \frac{37}{41}$. 6. $\frac{1}{2} - \frac{\sqrt{3}(x-\frac{\pi}{3})}{2} - \frac{(x-\frac{\pi}{3})^2}{4}$. 7. $y = -2x + 1$.
 8. $\frac{11}{36}$ 9. $y = 1$. 10. $y^2z^2 + 4y^3 + x^3y^2$.
 B11. (a) $x = r \cos \theta, y = r \sin \theta$; (c) AC^{-2}, C^{-1} .
 12. (b) $x = X_0 \cos \sqrt{\frac{k}{m}}t, y = V_0 \sqrt{\frac{m}{k}} \sin \sqrt{\frac{k}{m}}t$.
 13. (a) minimum at (0,0); (b) Saddles at $(\pm 4, 0)$, Minimum at (0,2), Maximum at (0, -2).
 14. (a) (i) 1.
 15. $\frac{\sin \mu\pi}{\mu\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{2\mu(-1)^n \sin \mu\pi}{\mu^2 - n^2} \cos nx; A = 3, B = 3$.
 16. (0, 2, 2), (0, 0, 0); F_1 : (a) 1, (b) $\frac{\pi}{2}$; F_2 : (a) 0, (b) 0.
 17. (a) (i) $\frac{16}{35}$; (ii) $\frac{7}{16}$; (b) $\frac{\binom{34}{3}\binom{64}{4}}{\binom{100}{7}}$; (c) (i) $\alpha = P(B|A)P(A), \beta = (P(B|A) - 1)P(A) + 1$;
 (ii) $\alpha = \text{Min}(P(B|A), P(B|\bar{A})), \beta = \text{Max}(P(B|A), P(B|\bar{A}))$.
 18. (a) 224; (b) $\frac{1}{2}(e - 1)$; (c) $\frac{81\pi}{4}$
 19. (a) $C = 6$; (b) $D = -1, E = 5$.
 20. (a) $H_0 = 1, H_1 = 2x, H_2 = 4x^2 - 2$.

2011

Paper I

A1. 1. 2. (a) $-\frac{1}{2}\ln(3-2x) + c$. 3. (a) (0,1); (b) $(e, e^{1/e})$.

4. (a) spiral, starting from $(0, 4\pi)$ and going clockwise into origin; (b) $7\pi/2$. 5. $\pi: 3\sqrt{3}$.

6. $\pm\frac{\pi}{6}, \pm\frac{5\pi}{6}$. 7. $\ln\frac{1}{4}$ or $\ln\frac{3}{4}$. 8. $e^{-x^2} \sin x$. 9. $\frac{1}{x \ln x}$. 10. (a) 10^6 ; (b) $\frac{8}{7}$.

B11. (b)(i) $\frac{1}{3} - \frac{x^2}{54}$; (ii) $3 \ln 2 + \frac{3x}{2} - \frac{3x^2}{8} + \frac{x^3}{8}$; (iii) $1 + x + \frac{1}{2}x^2$.

12. (a) $\frac{1}{s} e^{-\frac{s}{5}}$; (d) $\frac{\bar{t}}{s+\bar{t}}$; (f) $(1 - \frac{\bar{t}}{s+\bar{t}}) e^{-\frac{r_0}{s}}$; (h) \bar{s} .

13. (a) (i) 16; (ii) $\frac{1}{16} \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$; (iii) 2 and 8; $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$; (b) $b = 5, c = -6, d = 5$

(c) $2x'^2 + 8y'^2 = 1, B = \begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix}$.

14. (d) $a_m = b_m = 0$ if $m \neq 1, a_1 = -\sqrt{3}, b_1 = 1$; (e) $2 \sin(x - \frac{\pi}{3})$; (g) 4π .

15. (d) $s = \frac{[a,b,n]}{[b,m,n]}, t = \frac{[a,b,m]}{[a,m,n]}$.

16. (a) $y = 2 \ln x - 1$; (b) $y = (x^2 - 1)e^{-x^2}$; (c) $y = \frac{8}{3} \cos 2x - \frac{\sqrt{2}}{3} \sin 2x + \frac{1}{3} \cos x + \frac{1}{3} \sin x$.

17. (b) (i) $2, -\frac{\pi}{3}$; (ii) $\sqrt{5}, \tan^{-1} 2$; (iii) $\sqrt{2}, \frac{5\pi}{12}$; (b) $e^{\pm i\pi(\frac{2}{9} + \frac{2n}{3})}$.

18. (b) first not exact, second is exact (c) $\mu = 1/\sin x, y = -\frac{1}{x} \ln(\sin x) + \frac{c}{x}$; (d) $(\frac{\partial a}{\partial d})_e$.

19. (a) zero; (b) π (or $-\pi$, if you take $d\mathbf{S}$ to be in negative y direction); (c) $4\pi kr^3$; (d) $\pm 4\pi$.

20. (a) All cases continuous and differentiable except: $x = 0$ and $\pm\pi, f_0$ is continuous and differentiable, f_1 and f_2 are not even continuous (limit is non-zero but function is defined to be zero there).

At $\pm\frac{\pi}{2}$, I think the question is just a more tiresome version of the supervision sheet question Mich Term

Examples I J1. At $\pm\frac{\pi}{2}, f_0$ is not even continuous; f_1 is continuous but not differentiable; f_2 is continuous and differentiable. The idea of plotting f_1 and/or f_2 is horrendous. (b) $x^2 + \frac{1}{6x^8} + O(\frac{1}{x^{18}})$.

Paper II

A1. $c = 6$. 2. $\ln(x^2 + 3x - 2) + c$. 3. $x = -1, -5$, discontinuities at $x = -2, 1$. 4. -.

5. $Re = \frac{14}{25}, Im = -\frac{23}{25}$. 6. $1 + \frac{(x-1)}{2} - \frac{(x-1)^2}{8}$. 7. $y = -\ln(1 - \frac{1}{2}\ln(x^2 + 1))$.

8. Circle, radius 2, centre (0,1). 9. $y = 12x - 15$. 10. Zero.

B11. (a) (i) π ; (ii) Zero; (iii) Zero; (b) (i) and (ii) are both $\pm 10e\pi\mathbf{k}$ (don't know which until the rim traversal direction is specified?)

12. -.

13. (b) $-\sqrt{3}$.

14. (a) (i) 45; (iii) $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, anything satisfying $x + 2y - 3z = 0$, eg $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

15. (a) $x = \frac{V_{0x}}{\gamma} (1 - e^{-\gamma t}), z = (\frac{V_{0z}}{\gamma} + \frac{g}{\gamma^2}) (1 - e^{-\gamma t}) - \frac{gt}{\gamma}$;

(b) $z = \frac{g}{\gamma^2} \left[\ln\left(1 - \frac{\gamma x}{V_{0x}}\right) + \left(\frac{\gamma^2 V_{0z}}{g} + \gamma\right) \frac{x}{V_{0x}} \right]$.

16. (a) $E(\frac{2}{3}, \frac{\pi}{6})$; (c) $\frac{\pi}{2a} (3a^2 + b^2)$.

17. (a) $\frac{(32!)^3}{64!(16!)^2}$; (b) $l = \frac{3}{4}, p = \frac{4}{5}$.

18. (a) $\frac{R^3}{6}$; (b) $\frac{4}{3}$; (c) 8π .

19. (c) $\Psi(x, y) = \frac{2V}{\pi} \tan^{-1} \left(\frac{2y}{1-x^2-y^2} \right)$.

20. (a) $L_{10} = \frac{1}{10!} x^{10} - \frac{10}{9!} x^9 + \dots$.

2012

Paper I

- A1. $f = -1$ $x = 2$. 2. $y = -\frac{1}{4}x + \frac{13}{4}$. 3. $p = 7$; $x^2 + 2x + 4$ rem 5.
 4. (a) $(1 - 2x)e^{-2x}$; (b) $\frac{1}{x^2} - \frac{\ln x}{x^2}$. 5. $\pi^2 - 4$. 6. $\frac{1}{x^2-1} + \frac{2}{x-1} + \frac{2}{x+2}$. 7. $y = 3e^{x^3-1}$.
 8. (a) $\frac{1}{8}$; (b) $84x^3$. 9. - 10. $\frac{\pi^2}{2}$.
 B11. (a) $a = 0$ or $b = \pm c$; (b) $k(6 \ 1 \ -3)^T$; (c) $a = 0$ $b = -\frac{7}{3}c = \frac{5}{3}$ (d) no.
 12. (a) (i) $\text{Re} = -2$ $\text{Im} = \frac{3}{2}$; (ii) $\text{Re} = 0$ $\text{Im} = 32$; (iii) $\text{Re} = \frac{5}{4}$ $\text{Im} = 0$; (b) $\sqrt[4]{2}e^{(\frac{3\pi}{8}+n\pi)i}$;
 (c) $\frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$; (d) $z = (1 + 2n)i\pi$.
 13. (a) $y = -\frac{x}{\ln x+c}$ or $y = 0$; (b) $y = \frac{1}{x}(k - \frac{1}{2}x^2)$; (c) $y = \frac{a(k \sin mx - m \cos mx) + (k^2 + m^2 + am)e^{-kx}}{k^2 + m^2}$.
 14. (a) **(a.c)b - (a.b)c**; (b) lines parallel; (c) $\mathbf{a.c} = \mu$, $\mathbf{a.b} = \lambda$; (e) $\sqrt{\lambda^2/|\mathbf{b}|^2 + \mu^2/|\mathbf{c}|^2}$.
 15. (a) $(\frac{3}{4}, \frac{3\sqrt{3}}{4})$; (b) $\frac{3\pi}{2}$; (c) 8.
 16. (a) $-\frac{2}{5}$; (b) $(0, \pm\sqrt{2}), (\pm\sqrt{2}, 0)$; (c) saddle point.
 17. (a) 0 unless $m = n \neq 0$ in which case L , or $m = n = 0$ in which case $2L$;
 (b) $\frac{e-e^{-1}}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n(e-e^{-1})}{1+n^2\pi^2} (\cos n\pi x - n\pi \sin n\pi x)$.
 18. (a) (i) $(\frac{3}{4})^{24}$; (ii) $(\frac{1}{4})^{24}$; (iii) 18; (iv) $\frac{3}{2}\sqrt{2}$; (b) (i) $\int_{\alpha}^{\beta} f(x)dx$; (ii) $\int_{\alpha}^{\beta} xf(x)dx$, $\int_{\alpha}^{\beta} (x - \mu)^2 f(x)dx$;
 (c) (i) $A = \lambda^2$; (ii) $2/\lambda$, $2/\lambda^2$.
 19. (a) (ii) continuous and differentiable; (iii) zero; (iv) $\frac{(-1)^n}{n!}$; (b) (i) 2; (ii) limiting value = 2 always.
 20. (a) $h = 1$ $w = l = 2$; (b) $(\sqrt[4]{\frac{1}{2}}, \sqrt[4]{2} - \sqrt[4]{\frac{1}{2}})$.

Paper II

- A1. $a = 2$ $b = 1$ $c = 3$. 2. $\frac{1}{2}(e - 1)$. 3. $\frac{1}{5}(\begin{matrix} 4 & -1 \\ -3 & 2 \end{matrix})$. 4. $\text{Re} = -\frac{7}{41}$, $\text{Im} = \frac{22}{41}$.
 5. $\ln 4 + (x - 2) - \frac{1}{4}(x - 2)^2$. 6. $\omega = \frac{2}{3}$. 7. $\frac{2}{3}$. 8. $3x + 6z$, $(0, 2x, 3y)$. 9. $\frac{1}{2}$.
 10. $x = 0, \pm\sqrt{2 - \pi/2}, \pm\sqrt{2 + \pi/2}$.
 B11. (c) $\frac{1}{\sqrt{2}}(1 \ 0 \ 1)^T, (0 \ 1 \ 0)^T, \frac{1}{\sqrt{2}}(1 \ 0 \ -1)^T$ (f) $\mathbf{B}^{-1} = \mathbf{B}^T$
 12. (a) $\cos^k x(1 - \sin^3 x)^n, -\cos^k y(1 - \sin^3 y)^n, 0$; (b) $f(n) = \frac{3n}{3n+1}$; (c) $\frac{81}{140}$; (d) $\frac{1}{3(n+1)}$.
 13. (a) $y = \frac{1}{3}e^x + \frac{2}{3}e^{-2x}$; (b) $y = (A + Bx)e^{-3x} + \frac{x}{3} - \frac{1}{9}$; (c) $y = Ae^{-x} + Be^{-2x} - \frac{3}{4} - \frac{x}{2} + \frac{1}{20}e^{3x}$.
 14. (a) (i) $\sqrt{\frac{2}{3}}$; (ii) $A = \frac{\sqrt{6}}{2}, \hat{\mathbf{n}} = \frac{1}{\sqrt{6}}(1 \ 1 \ 2)^T, -\frac{1}{2}(1 \ 1 \ 2)^T$; (iii) $\frac{3}{\sqrt{6}}$; (b) 3.
 15. (a) $V = \frac{kr^2}{2a^3} + A, V = -\frac{k}{r} + B$; (c) $B = 0$ $A = -\frac{3k}{2a}$
 16. (a) $162(t \cos^4 t + \cos^3 t \sin t - 3t \cos^2 t \sin^2 t) + 9(\cos^2 t - \sin^2 t) + 2, -7$; (b) $a = 2$; (c)
 $\frac{x^3}{3} + \frac{x^2y}{2} - xy^2 = k$.
 17. (b) $\frac{23}{16}$; (c) $\ln 2 + \frac{x}{2} + \frac{x^2}{8}$.
 18. (a) $\frac{13}{18}, \frac{5}{9}, \frac{4}{5}, \frac{8}{13}, \frac{4}{9}$; (c) $\frac{1}{6}$; (d) $\frac{8}{17}$.
 19. (a) $f(a, a) + \int_0^a \frac{\partial f(x, a)}{\partial a} dx; -\frac{1}{a^2} + 2e^{-a^2} + \frac{e^{-a^2}}{a^2}$; (b) $\frac{1}{b+1}$.

20. (c) $u(x, t) = ax.$

2013

Paper I

- A1. (b) $\max y = \sqrt{2}$. 2. (a) $(\frac{1}{2}, \frac{1}{4})$; (b) $\pi/2$. 3. (a) $y = \ln(e^x + c)$; (b) $y = \ln(e^x + 1)$.
 4. $-\frac{1}{6}, 8.5$. 5. (b) 3. 6. (a) $\frac{x^{m+1}}{m+1} (\ln x - \frac{1}{m+1}) + c$; (b) $\frac{(\ln x)^2}{2} + c$. 7. (a) $\frac{5}{x-3} + \frac{2}{x+7}$; (b) $(x-3)^2 + 5$.
 8. (a) $a^x \ln a$; (b) $-\cos(\cos x) \sin x$. 9. Assuming $b(0) = 0$: (a) $\sqrt{100 + 16t^2}$; (b) $12/\sqrt{10}$. 10. $\frac{44}{15}$.
 B11. (a) $(1\ 2\ 3)^T$; (b) a and b axes plus circles radii 1,2,3... centred on origin.
 12. (a) $2, -1 \pm i\sqrt{3}$; (b) $2 + i$ or 1 ; (c) $-2, -i$; (d) $\frac{3}{t^2+1}; \frac{6}{(t^2+1)^{3/2}}$.
 13. (a) $y = (1+x^2)^{-2} (\tan^{-1} x + 1)$; (b) $x = y \ln y - 2 + cy$; (c) $y = \frac{1}{x^2(\sin x + c)}$.
 14. (c) $x - 4y - z + 5 = 0$; (e) $75\sqrt{2}/8$.
 15. (a) $az = r^2$; (b) $\frac{a^2\pi}{6} \left(\left(\frac{4h}{a} + 1 \right)^{\frac{3}{2}} - 1 \right)$; (c) (i) $\pi ah^2/2$; (ii) $(0, 0, 2h/3)$.
 16. (a) Saddles at $(0,0), (\pm 1,0), (0, \pm 1)$, minima at $\pm (\frac{1}{2}, \frac{1}{2})$, maxima at $\pm (\frac{1}{2}, -\frac{1}{2})$;
 (b) (ii) $y = \sin 2x \cos 5t$.
 17. (a) $L \left(\frac{a_0^2}{2} + \sum a_n^2 + b_n^2 \right)$; (b) $\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{2}{(2n-1)^2\pi} \cos(2n-1)x - \frac{\sin nx}{n}$
 18. (a) (i) $\frac{100}{231}$; (b) (i) $\frac{2}{5}$; (ii) $1 - 0.5\alpha$; (iii) $\frac{0.8-0.2\alpha}{2-\alpha}$.
 19. (b) $h > f > g$; (d) diverges if $p \leq 1$ converges if $p > 1$.
 20. (a) $\frac{4a^3}{3\sqrt{3}}$.

Paper II

- A1. (a) $(-4\ -9\ 13)^T$; (b) -4 . 2. (a) 1; (b) $\frac{u}{\sqrt{1-u^2}}$. 3. $-\frac{5}{13}, -\frac{12}{13}$. 4. $\frac{7}{\sqrt{5}}$. 5. 0; $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $5: \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
 6. $\frac{1}{2} - \left(x - \frac{\pi}{4}\right)^2$. 7. (a) $r = \sqrt{3} \phi = \frac{\pi}{3} \theta = \frac{\pi}{6}$. 8. (a) $(\cos xy + xy \cos^2 xy - xy \sin xy)e^{\sin xy}$;
 (b) $2xz + 3y^2z$. 9. (a) $-\frac{r}{r^3}$. 10. (a) $\frac{1}{216}$ (b) $\frac{91}{216}$.
 B11. (b) $\frac{\alpha}{\alpha-1}, \alpha > 1$; (c) $\frac{\alpha}{(\alpha-2)(\alpha-1)^2}, \alpha > 2$; (d) $F(x) = 1 - x^{-\alpha}, x = 2^{1/\alpha}$; (e) $\frac{1-3^{-\alpha}}{1-6^{-\alpha}}$.
 12. (a) (i) $(\mathbf{a} \cdot \mathbf{b})^2$; (ii) $\mathbf{a} \cdot \mathbf{b} + |\mathbf{a}|^2 |\mathbf{b}|^2$; (iii) $(\mathbf{a} \cdot \mathbf{a})^6$; (b) (i) zero; (ii) $\begin{pmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}$; (iii) $\begin{pmatrix} -\frac{1}{4} & 0 \\ 0 & -\frac{1}{4} \end{pmatrix}$,
 $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$; (iv) $\begin{pmatrix} 0 & -\frac{1}{8} \\ \frac{1}{8} & 0 \end{pmatrix}$; (v) $\frac{(-1)^n}{2^{2n}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \frac{(-1)^n}{2^{2n+1}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.
 13. (a) $\sqrt{\pi}$; (b) (i) $I_n = \frac{n-1}{2} I_{n-2}$; (ii) $I_1 = 0, I_2 = \frac{1}{2} \sqrt{\pi}$ (c) (ii) zero; (d) $\frac{\pi}{3\sqrt{3}}$.
 14. (a) $y = -\frac{1}{3}x^3 - \frac{1}{3}x^2 - \frac{8}{9}x + d + ce^{3x}$; (b) (i) $y = A \cos 2x + B \sin 2x + \frac{1}{3} \sin x$;
 (ii) $y = A \cos 2x + B \sin 2x - \frac{x}{4} \cos 2x$; (iii) $y = A \cos 2x + B \sin 2x + \frac{1}{3} \sin x - \frac{x}{4} \cos 2x$;
 (c) $y = 3e^t - e^{2t}$.
 15. (a) $(a, 0)$; (b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; (c) $y \rightarrow \pm \frac{bx}{a}, x \geq a$; (d) difference in distances is $2a$.
 16. (a) (i) $(6x^2\ 2y - 1\ 6z)^T$; (ii) $\frac{1}{\sqrt{2}}(1\ 1\ 0)^T$; (iii) $\frac{27}{\sqrt{2}}$; (iv) $\int_0^{2\pi} V d\theta = 62\pi$.
 17. (a) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T, \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T, \left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial T}{\partial P}\right)_S$; (b) $U = cT - \frac{a}{V}$.

18. (b) $(0,0)$; (c) $x \geq 0, y = x^2 \pm x^{5/2}$; (d) Min and zero on both branches at $x = 0$, max on negative branch at $x = 16/25$, zero on negative branch at $x = 1$.

19. (b) $-a^2\pi$.

20. (a) $f(b(\theta), \theta) \frac{db}{d\theta} - f(a(\theta), \theta) \frac{da}{d\theta} + \int_{a(\theta)}^{b(\theta)} \frac{\partial f}{\partial \theta} dx$; (c) $F(t) = \sqrt{\frac{\pi}{2}} e^{-t^2/2}$.

2014

Paper I

- A1. (a) $-\frac{2x}{(x^2+4)^2}$; (b) $\cos x e^{\sin x}$. 2. (a) $-a^{-x} \ln a$; (b) $\ln x \ln(\ln x) - \ln x + c$. 3. (a) $\frac{1}{2} \ln 2$; (b) $-\ln 2$.
4. (a) $\tan y = c - \cos x$; (b) $y = 3e^{3x}$. 5. (a) $(\frac{4}{3}, \frac{7}{3})$ and $(-1, 0)$; (b) ellipse centre $(1, 0)$.
6. (a) $1/40\pi$; (b) $1/3\sqrt{2}$. 7. -. 8. (a) $x = \frac{7}{3}$ or -1 ; (b) Min. 9. $\frac{2}{3}$. 10. $y = -\frac{1}{2}x - \frac{3}{4}$; $\frac{5}{x-4} + \frac{8}{x+9}$.
- B11. (a) $\text{Det } A=0, \text{Tr } A=6$, at least one eigenvalue zero; (b) $\lambda = 0 \mathbf{x} = (1 \ 2 \ 1)^T$, $\lambda = -6 \mathbf{x} = (1 \ 0 \ -1)^T$, $\lambda = 12 \mathbf{x} = (1 \ -1 \ 1)^T$; (c) $\mathbf{e} = k(1 \ 2 \ 1)^T$; \mathbf{Ar} lies in the plane $\perp (1 \ 2 \ 1)^T$.
12. (a) modulus $= \sqrt[6]{2}$, $\arg = \frac{\pi}{4} + \frac{2n\pi}{3}$; (b) $z = i(-\frac{\pi}{4} + n\pi)$; (c) $1, (1+i)$.
13. (a)(i) $y = \frac{4}{3}e^{-3x} + \frac{8}{3}$; (ii) $y = e^{\sin x} - (1 + \sin x)$; (b) $(2+2x)e^{-3x} - e^{-4x}$.
14. (a) $(\mathbf{a} - \mathbf{c}) \cdot (\hat{\mathbf{b}} \wedge \hat{\mathbf{d}}) / |(\hat{\mathbf{b}} \wedge \hat{\mathbf{d}})|$; (b) Quick: just say it's a plane, slower $\mathbf{p} \cdot \mathbf{q} \neq 0: \lambda \mathbf{p} + (k - \lambda \mathbf{p} \cdot \mathbf{p})/\mathbf{p} \cdot \mathbf{q} + \nu \mathbf{p} \wedge \mathbf{q}, \mathbf{p} \cdot \mathbf{q} = 0: k\mathbf{p}/\mathbf{p} \cdot \mathbf{p} + \mu\mathbf{q} + \nu \mathbf{p} \wedge \mathbf{q}$; (c) $r = 1, \phi = t, \theta = \cot^{-1} t$.
15. (a)(ii) $\frac{9\sqrt{3}}{8} - \frac{\pi}{4}$; (iii) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$; (b) $x_0(a\sqrt{\pi})^3$.
16. (a) Saddles at $\pm(3, 4)$, minimum at $(5, 0)$, maximum at $(-5, 0)$; (b) Saddle at $(0, 0)$, minima at $\pm(3, 3)$.
17. (b) $\cosh \pi$.
18. (c) (i) $\frac{28}{45}$; (ii) $\frac{16}{45}$; (iii) $\frac{1}{45}$; mean $= \frac{2}{5}$; var $= \frac{64}{225}$.
19. (a) $|x| < 2$, diverges at $x = -2$, converges (conditionally) at $x = 2$; (b) $1 - \frac{3x}{2} + \frac{11}{8}x^2$; (c)(i) diverges; (ii) converges; (d) $S(x) = (x^3 - 2x^2 + 2x)e^x - 2x$.
20. (a) $\sqrt{8}$.

Paper II

- A1. (a) $a = 1, b = \frac{4}{3}$. 2. (a) $\text{Re} = \cosh \alpha \cos \beta, \text{Im} = \sinh \alpha \sin \beta$. 3. $x + \frac{x^2}{2}$. 4. $y = Ae^x + Be^{3x}, y = x + \frac{4}{3}$. 5. $\text{div} = -e^{-x} \cos z - e^{-y} \sin z, \text{curl} = (-e^{-y} \cos z \ -e^{-x} \sin z \ 0)^T$.
6. $u = \cos(x - ct)$. 7. e^ϕ and $e^{-\phi}$. 8. $(\pi, \frac{\pi}{2}), (\frac{\pi}{2}, \pi), (\pi, \frac{3\pi}{2}), (\frac{3\pi}{2}, \pi)$. 9. (a) zero; (b) $-\frac{2}{3}$.
10. (a) 1 (b) $2^{-\alpha} - 3^{-\alpha}$.
- B11. (a) $\frac{p_0}{p_0+p_1}$; (b) $A = \sigma^{-2}$, mean $= \sigma \sqrt{\frac{\pi}{2}}$; (c) $\binom{m+n}{m} \theta^m (1-\theta)^n$.
12. (a) (ii) $\mathbf{M}^{-1} = \frac{1}{8} \begin{pmatrix} 3 & 2 & 3 \\ 1 & -2 & 1 \\ -4 & 0 & 4 \end{pmatrix}$, no; (b) (ii) $x = -2, y = 3, z = 2$; (iii) $a = 1$ or $a = 3$
- (iv) $z = 1, x + y = 2$.
13. (a)(i) $x + e^x + c$; (ii) $\frac{1}{6} \ln(x^2 + \frac{2x}{3} + \frac{2}{3}) - \frac{4}{3\sqrt{5}} \tan^{-1} \frac{3x+1}{\sqrt{5}} + c$; (b) zero; (c) $f(x)$; (d) $2^N \sin(x^2 + x^6)$.
14. (a) $f = 2x^2 + \frac{x^2 y^2}{2} + \frac{y^2}{2} + c, y = \sqrt{\frac{12-4x^2}{1+x^2}}$; (b) $f(x) = \frac{1}{N} (\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}), \ln \psi = \int \frac{1}{M} (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}) dy$; (c) $x^3 y^2 + x^2 y = k$ and hence $y = \frac{-x^2 \pm \sqrt{x^4 + 4kx^3}}{2x^3}$.
15. (b) $u_n = (1)(-2)(-5) \dots (4-3n)x^n / 3^n n!$; (c) $f(x) = \frac{1}{\sqrt{2}} - \frac{\sqrt{2}x}{\pi} + \frac{\sqrt{2}}{\pi^3} (4-\pi)x^2 + \dots$; (d) $\ln x^2 + \frac{1}{x} + \frac{1}{2x^2}$.

16. (a)(i) 0; (ii) 0; (b) only \mathbf{F}_1 is conservative, $\phi = x^3 y^2 z + c$, 2.

17. (a) $dp = \left(\frac{RT}{V} - \frac{2na}{V^2}\right) dn + \left(\frac{2n^2 a}{V^3} - \frac{nRT}{V^2}\right) dV + \frac{nR}{V} dT$; (b) $\hat{\mathbf{r}}g'(r)$; (c)(i) 1; (ii) $\frac{1}{\sqrt{14}}(2 \ -3 \ 1)^T$.

18. (a) Maxima at $x = n\pi/2$ n odd, minima at $x = n\pi/2$ n even;

(c) $\frac{1}{2y\sqrt{\ln y}}$, $\frac{-1}{4y^2}(\ln y)^{-\frac{3}{2}}(1 + 2 \ln y)$; (d) $\cos x$; (e) $-\sin x$; (f) $-\sin x$.

19. $c^2 = \frac{T}{\rho}$, $y = \frac{2}{c} \sin \frac{ct}{2} \sin \frac{x}{2} + \frac{2}{5c} \sin \frac{5ct}{2} \sin \frac{5x}{2}$.

20. (b) $\frac{1}{x^2} \cos x^3 + 3x \sin x^3 - \frac{1}{x^2} \cos 3x^2 - 6 \sin 3x^2$; (c) $I'(a) = \frac{1}{1+a^2}$, $I(a) = \tan^{-1} a$, $I(1) = \frac{\pi}{4}$.