

## M2 2007 Section A cribs

### Short Answers

**Only Brief answers but not details are given in this document!  
You need to provide details in your answers!**

1(a)

See the lecture notes for the definition of the law of averages. Example 1: A drunk.  
Example 2: A pie-vendor inventory problem.

1(b)

Possible approaches are: Diversification, Real options; Portfolio analysis; System redesign; .....

1(c) (i)

A 95% confidence interval for the population percentage is  $[Q-2*STEP, Q+2*STEP] = [57.80, 85.06]$ , which does contain the population percentage 65%. Therefore, .....

NOTE: one may get a different conclusion if one uses a different confidence level.

1(c)(ii)

A 95% confidence interval the population percentage is  $[m-2*STEM, m+2*STEM] = [444.85, 559.14]$ , which does contain the population mean 450 miles. Therefore, .....

1(c)(iii)

By CLT, the sample mean RV  $X$  has a normal distribution  $N(\mu, \sigma/\sqrt{n})$ .  
The smallest significance level at which the average time per day staying in the vehicle for all drivers in the state of New Mexico is significantly different from the national average is  $2*P(X \geq m) = \dots = 2*p(z \geq 1.16667) = 0.243345$ . Therefore, this smallest significance level should be about 24.3%.

2(a)

$$Y = 19.475 - 3.083 X_1 + 10.944 X_2 + 0.288 X_3$$

Interpretations ....

2(b)

R-square statistic = 0.598. See the lecture notes for the definition of R-square statistic.

2(c)

When  $X_1 = 3$ ,  $X_2 = 4$ ,  $X_3 = 5$ ,  $Y = 55$ , a 95% CI is  $[Y - 2 * Sterr, Y + 2 * Sterr] = [43.85.00, 67.02]$ . Interpretations .....

2(d)

When  $X_1 = 3$ ,  $X_2 = 4$ ,  $X_3 = 5$ , a 95% CI for the average Usage level is  $[Y - 2 * Sterr / \sqrt{n}, Y + 2 * Sterr / \sqrt{n}] = [54.28, 56.60]$ . Interpretations .....

2(e)

A 95% CI for the slope of Price level is  $[-4.466, -1.700]$ . A rough 99% CI for the slope of Overall service level is  $[b - 3 * Sb, b + 3 * Sb] = [8.00, 13.88]$ .

2(f)

The impact of one unit increase in Product quality level is 0.288, and with a 95% CL, the impact on Usage level is between  $[-0.609, 1.185]$ . So the impact from two units increase in Product quality is  $2 * [-0.609, 1.185] = [-1.217, 2.370]$ . Explain the result intuitively again.

2(g)

Check R-square statistic, standard errors, p-values and confidence intervals etc. Make some recommendations according to your observations. We need to re-do regression by dropping Quality level .....

3(a)

Draw a decision tree. The EMVs for three options “settle completely”, “negotiate patiently” and “fight in court directly” are \$4.8 billion, \$3.44 billion, and \$6.04 billion, respectively. Obviously, the best decision would be .....

3(b)

There are three possible outcomes if CellPower chose the negotiation option (and chose to the fight option in the next stage): \$200 million, \$8 billion, and \$2 billion, respectively. The probabilities for those outcomes are 20%, 30% and 50%, respectively.

3(c)

This is about sensitivity analysis. Yes, CellPower will change its decision when the probability to lose in option “fight in court directly” is reduced. This happens when the EMVs are the same for both “negotiate patiently” and “fight in court directly” options. Therefore we need to find a probability to lose in the “fight in court directly” option such that two EMVs are equal:  $3.44 = p \cdot 10 + (1-p) \cdot 0.1$ . Hence  $p = 0.337$  approximately. Therefore .....

3(d)

If CellPower is conservative, they may choose the worst-case scenario for their decision. Re-evaluate the decision tree based on this new criterion. The worst values for three options “settle completely”, “negotiate patiently” and “fight in court directly” are \$4.8 billion, \$5 billion, and \$10 billion, respectively. In this case, CellPower would choose .....

4 (a)(i)

Define decision variables

X1 – The number of people surveyed who are 30 years of age or under and live in urban areas

X2, X3, Y1, Y2, Y3 etc

The overall LP is

Min  $7.00 X1 + 7.20 X2 + 6.00 X3 + 7.50 Y1 + 7.80 Y2 + 6.50 Y3$

s.t.  $X1 + X2 + X3 + Y1 + Y2 + Y3 \geq 2,000$

$X1 + Y1 \geq 1,000$

$X2 + Y2 \geq 500$

$Y1 + Y2 + Y3 \geq 25\% (X1 + X2 + X3 + Y1 + Y2 + Y3)$

$X1 \geq 20\% (X1 + X2 + X3 + Y1 + Y2 + Y3)$

$Y3 \leq 20\% (X1 + X2 + X3 + Y1 + Y2 + Y3)$

$X1, X2, X3, Y1, Y2, Y3 \geq 0.$

Explain each term above .....

4(a)(ii)

Define a binary variable Z such that  $Z = 1$  if and only if the number of people surveyed from rural areas is greater than or equal to 500, i.e.,  $Y1 + Y2 + Y3 \geq 500$ . We need to add the following constraints:

$500 Z \leq Y1 + Y2 + Y3 \leq 499 + \alpha Z$  (alpha is a big positive number)

Z is binary

X1, X2, X3, Y1, Y2, Y3 are positive integers.

The new objective function is

$7.00 X1 + 7.20 X2 + 6.00 X3 + 7.50 Y1 + 7.80 Y2 + 6.50 Y3 + 1000 Z$

4(b)(i)

If the unit cost for variable “Make Model 2” increases by 10 units, while all other parameters are unchanged, the optimal solution remains unchanged according to the sensitivity report as 10 units is within the maximum allowable increase, which is 14, for the objective coefficient of “Make Model 2”. The change of the objective function value would be  $550 \times 10 = 5,500$  units.

4b(ii)

If you increase resource “Harnessing” by 100 units, while other parameters are unchanged, the change of the optimal objective function value would be  $100 \times (-7) = -700$  units because .....

4(b)(iii)

If you increase resource “Wiring” by 10 units, while other parameters are unchanged, the change of the optimal objective function value would be 0 because .....

## Brief answers to the questions on the M2 Section B 2007

**Note: Only brief answers—not details—are provided here. In your test paper, you will need to provide details when answering each question!**

### Question 5.

- (a) Opportunity cost of capital tied up in inventory.
- (b) The Wagner-Whitin algorithm.
- (c) See the class notes.
- (d)  $Q^c = \text{square-root} [2 (25/150)C_O D /C_H]$ . Thus,  $(Q^* - Q^c)/Q^* = 0.408 Q^*$ . It follows that average inventory level will decrease by 59.2%.
- (e) The ordering frequency will increase by a factor of  $Q^*/Q^c = \text{square-root} [6]$ . Thus: Order frequency will increase by a factor of 2.45.

### Question 6.

- (a) (1) Excess overtime costs. (2) Long truck queues waiting to unload berries that develop during the peak season.
- (b) A bottleneck in the process flow.
- (c) The problems that we are trying to mitigate *only occur* on peak days.
- (d) Adding dryer capacity will have no effect on overtime or productivity. However, additional dryers will have an effect on *both* these problems.

### Question 7.

Yes, there is sufficient information.

The average flow rate for the system as a whole is 5 per minute. The average number of people waiting or being served at the tills is 20 people. Applying Little's Law, the average flow time at the tills is 4 minutes.

The average flow rate for the sandwich counter is 2 per minute. At the sandwich bar, the average number of people in queue waiting or being served is 6. Again applying Little's Law, the average flow time at sandwich counter is 8 minutes.

Thus, the total amount of time to get a sandwich and pay for it at the tills is 12 minutes.

### Question 8.

(a) The goal of a corporation is to make money.

(b) The three measurements are *Throughput*, *Inventory*, and *Operational expense*.

(c) When the wrong value of Q used is  $Q^W = (1/4) Q^*$ , then simple algebra yields:  $[TC(Q^W) - TC(Q^*)]/TC(Q^W) = (9/8) TC(Q^*)$ , which is a 12.5% increase.

(d) What probably happened is that the cost of the labour involved in a setup was included in the order cost  $C_O$ . However, for non-bottleneck resources where there is extra time, the marginal value of the time is zero, reducing  $C_O$  and so the EOQ.

(e) The large size of the production runs (e.g., Bucky Burnside's order for 1000 Model 12's) supports the conclusion that the dominant type of production is batch.