2011

A1. (i) $-1 \le x \le 1$; (ii) $\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}}$.

2. (i) $\begin{bmatrix} \alpha(\alpha-1)K^{\alpha-2}\ln(L+\beta) & \alpha K^{\alpha-1}(L+\beta)^{-1} \\ \alpha K^{\alpha-1}(L+\beta)^{-1} & -K^{\alpha}(L+\beta)^{-2} \end{bmatrix}$.

3. (i) $\alpha\beta \neq 3$; (ii) $-\sqrt{2} < \theta < \sqrt{2}$.

4. (i) $y = 500e^{-\alpha t}$; (ii) 6.93.

B5. (i) $\frac{\partial u}{\partial x} = 1 + 2\sqrt{\frac{y}{x}}, \frac{\partial u}{\partial y} = 2\left(\sqrt{\frac{x}{y}} + 2\right)$; (ii) $1 + 2\sqrt{\frac{y}{x}} = 3\lambda$, $\sqrt{\frac{x}{y}} + 2 = 3\lambda$, $3x + 6y = 30$; (iii) $(x,y) = \left(\frac{10}{3}, \frac{10}{3}\right)$; you might also get $\left(\frac{20}{3}, \frac{5}{3}\right)$ but that does not solve the first order equations unless you take the square roots that appear at various points as negative.

6. (i) $\begin{bmatrix} 1-c-a+ct & b \\ \alpha & -\beta \end{bmatrix}\begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} \bar{C}+\bar{I}+\bar{G} \\ M_s \end{bmatrix}$; (ii) $-\beta(1-c-a+ct)-\alpha b$ (iii) $Y = \frac{\beta(\bar{C}+\bar{I}+\bar{G})+bM_s}{\beta(1-c-a+ct)+\alpha b}, r = \frac{\alpha(\bar{C}+\bar{I}+\bar{G})-(1-c-a+ct)M_s}{\beta(1-c-a+ct)+\alpha b}$; $\frac{\partial r}{\partial \bar{I}} = \frac{\alpha \bar{I}}{\beta(1-c-a+ct)+\alpha b}$, so r falls when $\bar{I}$ falls; (iv) β/b.

C7. (a) results are significant: sample is 1.98 stdevs from mean, critical one tailed value is 1.645; (b) sample statistic is 1.543, claim is supported at 10% (critical 1.282) but not 5% (critical 1.645).

8. (a) -2; (b) N=81.

9. (a) $\alpha = 8.59, \beta = 0.606$; (b) 21; (c) 389.

10. -

D11. (c) It appears the assumption is $\lambda = 1$, with their $\Delta\pi_t$ being $\pi_t - \pi_{t-1}$; (d) $\mu_0 = 6.91$; (e) $\mu_0 = 6.91, 7.23$; (f) Assuming we use $t_{35} = 2.03$ rather than 1.96 standard deviations then 7.3 ± 3.1, actual value of 12% is 1.56 stdevs above expected, so still plausible.

12. (a) 8; (c) $E(X) = E(Y) = 1, E(XY) = \frac{5}{4}; Covar = \frac{1}{4}$; (d) $E(X|Y=2) = \frac{3}{2}, var(X|Y=2) = \frac{1}{4}$.

2012

A1. (a) $\alpha \geq 0$ apart from $\alpha = 1$ where fn is not defined; (b) $U = \ln w + c$; (c) 12.

2. (a) $x = 1, y = 0, U = 1$; (b) $x = 4, y = 17, z = 3, U = \frac{25}{4} + \ln 4$.

3. (a) Det is $15 + 2\alpha$. For $\boldsymbol{A}$ to be positive definite: if you say non-symmetric matrices can't be positive definite then $\alpha = -2$, if you apply the determinant test anyway then $\alpha > -\frac{15}{2}$, if you apply a more sophisticated test (beyond the scope of the course) for $\boldsymbol{x^T A x} > 0$ then $2 - \sqrt{60} < \alpha < 2 + \sqrt{60}$; (b) Yes.

4. (a) $\ln x = (x - 1) - \frac{(x-1)^2}{2}$; (b) Yes; (c) $y(t) = t^2 - t$.

B5. (a) Invest entirely in asset with largest expected return;
(c) $\delta^* = 0.348, \frac{\partial \delta^*}{\partial \sigma_2^2} = \frac{(1-\delta^*)}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$; asset 2 is becoming more risky and our utility fn says we do not like that.

6. (a) $Q = 50\frac{2}{3}, \pi_D = \pi_F = 80.2$; (b) $\pi_i = \frac{(100 - q_i - (n-1)\bar{q})q_i}{8} - 3q_i$;
(c) $p = 12.5 - \frac{76n}{8(n+1)}, \pi_i = \frac{722}{(n+1)^2}$, as $n \to \infty, p \to 3, \pi_i \to 0$.

C7. (a) 0.52; (b) 49/52; (c) 9/16.

8. (a) $c_1 = -3, c_2 = 2$; (b) $x < 0{:}\ 0, 0 \leq x \leq 1{:}\ 2x - x^3, x > 1{:}\ 1$; (c) 1/8.

9. I've assumed sample is large enough to approximate the t-distribution t=1.711 by the Normal distribution Z=1.645. That's actually somewhat approximate. (a) critical *y* is 9.1775; (b) $\mu = 9{:}\ 0.64, \mu = 10{:}\ 0.05, \mu = 11{:}\ 0.0001$; (c) power will increase when $\mu = 9$, be unchanged when $\mu = 10$, and decrease when $\mu = 11$.

10. (b) sample $Z = 1$, critical $Z = 1.96$, do not reject H_0; (c) 1.015 ± 0.029.

D11. (a) $E(X) = \frac{\theta}{2}, Var(X) = \frac{\theta^2}{12}$; (b) No, unbiased estimator is *twice* the sample mean;
(c) $\hat{\theta} = 2\bar{x}$ is consistent; (d) Variance of smaller sample is twice that of the larger sample, so you'd prefer the larger sample.

12. (d) $25k; (e) $H_0{:}\ \beta_0 = 0, H_1{:}\ \beta_0 \neq 0$, sample *Z*=-0.92, critical *Z*=1.96, do not reject H_0.

2013

A1. (a) convex; (b) neither concave nor convex.

2. (a) $c_1 = 1/(1+\beta^{3/2})$, $c_2 = \beta^{3/2}/(1+\beta^{3/2})$; (b) $c_1 = 0.74, c_2 = 0.26, z = 0, u = 1.17$.

3. (a) $\begin{pmatrix} 4 & 4 \\ 4 & y^2 \end{pmatrix}$, det = $4y^2 - 16$; (b) $(0,0)$ saddle, $\pm(\sqrt{12}, -\sqrt{12})$ minima.

4. (a) $1 + 3x + \frac{9}{2}x^2$; (b) $(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$; (c) $4 + \frac{3}{4}(x-3) + \frac{21}{8}(y-1)$.

B5. (a) $\begin{cases} Q = 40 - 2p & 0 \le p \le 15 \\ Q = 25 - p & 15 \le p \le 25 \\ Q = 0 & p \ge 25 \end{cases}$;

(b) $\begin{cases} q_D = 15 - \frac{\alpha}{2}, q_F = 5 - \frac{\alpha}{2}, p = 10 + \frac{\alpha}{2}, \pi = (10 - \alpha/2)(20 - \alpha) & 0 \le \alpha \le 7.93 \\ q_D = \frac{25-\alpha}{2}, q_F = 0, p = \frac{25+\alpha}{2}, \pi = \left(\frac{25-\alpha}{2}\right)^2 & 7.93 \le \alpha \le 25 \end{cases}$;

(c) $\frac{d\pi}{d\alpha} = -7.5$; (d) $q_D = 14, q_F = 4, p = 11, \pi = 18$, so profits down from 56.25 in (b).

6. (a) $x_t = 10 + 1.05x_{t-1}$; (b) $x_t = -200 + (1.05)^{t-2000}(210)$; (c) $a = 0.9, b = 5$, $y^* = 50$, y_t will converge; (d) $y_t = 0.85y_{t-1} + 10/3$, $y^* = 22.2$, system still converges.

C7. (a) 40%; (b) 4/5; (c) 1/2.

8. (a) $c = 1/6$; (b) -1/6; (c) 23/45.

9. (a) both zero; (b) no.

10. (a) $\hat{\beta} = \frac{\sum y_i \sqrt{x_i}}{\sum x_i}$.

D11. (a) test statistic = 0.5, critical value = $\pm$1.96, do not reject null; (b) 0.17;
(c) if you don't round intermediate results you'll get test statistic = 3.612 whereas if you do you'll get 3.62, critical value = $\pm$1.96, reject null.

12. (b) 3.6; (c) test statistic = 1.047, critical value = 1.684, do not reject null; (d) 472;
(e) test statistic = 2.325, critical value $\pm$2.000, reject null.

2014

A1. (a) continuous and differentiable, derivative = 0; (b) continuous but not differentiable.

2. (a) $x_1 = \frac{b_1(m-a_2p_2)+p_1b_2a_1}{p_1(b_1+b_2)}$, $x_2 = \frac{b_2(m-a_1p_1)+p_2b_1a_2}{p_2(b_1+b_2)}$.

3. (a) $y_t = 1.005^t(10100) - 10000$, $y_t = £3623$; (b) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$.

4. (a) $\alpha \geq 1$; (b) convex.

B5. (a) $\ln w = \frac{Y(1-\delta)+\beta Z}{(1-\alpha)(1-\delta)-\gamma\beta}$, $\ln c = \frac{Z(1-\alpha)+\gamma Y}{(1-\alpha)(1-\delta)-\gamma\beta}$; (b) $\frac{(1-\alpha)(1-\delta)-\gamma\beta}{w(1-\delta)}$
(c) $\alpha + \beta = 1, \gamma + \delta = 1$, determinant is zero so no solutions or (if $\frac{Y}{Z} = -\frac{\beta}{\gamma}$), infinitely many.

6. (a) homogeneous degree 1; (b) $\frac{dK}{dL} = -\frac{(1-\delta)}{\delta}\left(\frac{K}{L}\right)^{\rho+1}$, isoquants are convex; (c) $\frac{1}{\rho+1}$;
(d) straight line, slope $-(1-\delta)/\delta$.

C7. (a) 0.5; (b) (0.4, 0.6) for *X* = (1,2), (0.4, 0.6) for *Y* = (1,2); (c) (0.75, 0.25) for *X* = (1,2); (d) No.

8. (a) 51/990.

9. (a) 5.035; (b) just swap *x* and *y* in formulae (c) slope = 0.191, intercept = 11.36.

10. (a) True; (b) True; (c) True.

D11. (a) $(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$: Expectation = $(\beta, \beta, N\beta)$, Variance = $\left(\frac{\sigma^2}{\sum x_i^2}, \frac{N\sigma^2}{(\sum x_i)^2}, \sigma^2 \sum \frac{1}{x_i^2}\right)$

12. (b) test statistic = 3.73, reject H_0; (c) wage = 10.145 + 0.031exper;
(d) wage = 8.130 + 0.031exper; (e) 2.015.

2015

A1. (a) $e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$; (b) $2 + \frac{x}{4} - \frac{x^2}{64}$; (c) If you read the question literally: 2.002500$\pm$0.000002, or if they meant you to take an *extra* term for the error then 2.002498438$\pm$2x10^{-9}.

2. (a)(i) $bp_t + dp_{t-1} = a + c$; (ii) $p^* = \frac{a+c}{b+d}$; (iii) $p_t = \left(-\frac{d}{b}\right)^t p_0 + \frac{a+c}{b+d}\left(1 - \left(-\frac{d}{b}\right)^t\right)$; (iv) $|d| < |b|$; (b)(i) $\frac{dy}{dt} + by = 0$; (ii) $z_t = (z_0 - z_m)e^{-\beta t} + z_m$.

3. (a) $e^5 - e^0$; (b) $\frac{11e^{12}}{9} - \frac{2e^3}{9}$.

4. (a) Min at $(4,2)$, max at $(-4,-2)$; saddles at $(4,-2)$ and $(-4,2)$;
(b) saddle at $(0,0)$, max at $\left(1, -\frac{3}{2}\right)$.

B5. (a) $\alpha + \beta < 1$; (b)(i) $C^* = 2(rw)^{\frac{1}{2}}Q^{1/2\alpha}$; (ii) $\frac{dC^*}{dQ} = \frac{(rw)^{\frac{1}{2}}}{\alpha}Q^{\frac{1}{2\alpha}-1}$; (iii) they are equal; (iv) $C = r\overline{K} + \frac{w}{\overline{K}}Q^{1/\alpha}$; (v) $K = \left(\frac{w}{r}\right)^{\frac{1}{2}} Q^{1/2\alpha}$.

6. (a) $1 + 2q, \frac{\alpha^2}{q} + 1 + q$; (b) $q = \frac{p-1}{2}$; p=35, q=17; (c) $\frac{d\pi^*}{d\alpha} = -2\alpha$; (d) $p = \frac{104+N}{N+2}, q = \frac{51}{N+2}$; (e) $N = 100, p = 2, q = \frac{1}{2}$; (g) $q = \frac{51}{4}, p = \frac{157}{4}$.

C7. (a) 0.6; (b) 0.75; (c) $\frac{3(2)^{n-1}}{5^n}$.

8. (a) $\mathrm{E}(X) = \frac{1}{\lambda}, \mathrm{E}(X^2) = \frac{2}{\lambda^2}$, var = $\frac{1}{\lambda^2}$.

9. (a) 10.7 ± 0.6; (b) $Z = 2.05$, reject H_0

10. (a) $\widehat{\beta_0} = 200, \widehat{\beta_1} = 0.82$.

D11. (c) $n = 5$: (d) $\frac{11}{2^{10}}$; (e) $(1-p)^{10} + 10(1-p)^9 p$, max at $p = 0$.

12. (a) $Z = -1.23$, do not reject H_0; (b) $Z = 3.17$, reject H_0;
(c) $Z = 11.7$, significantly different from zero.