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The economics of vaccination

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HIGHLIGHTS

- Several economic models of vaccination are analyzed.
- Equilibrium vaccination is compared to socially optimal vaccination.
- Features of diseases, vaccines and markets that lead to inefficiencies are identified.

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ABSTRACT

The market for vaccinations is widely believed to be characterized by market failures, because individuals do not internalize the positive externalities that their vaccination decisions may confer on other individuals. Francis (1997) provided a set of assumptions under which the equilibrium vaccination pattern is socially optimal. We show that his conditions are not necessary for the welfare theorem to hold but that in general, the market yields inefficiently low vaccination uptake. Equilibrium non-optimality may obtain if (i) agents can recover from infection, (ii) vaccines are imperfect, (iii) individuals are ex ante heterogeneous, (iv) vaccination timing is inflexible or (v) the planning horizon is finite. Apart from the case with heterogeneity, inefficiencies result from the presence of strategic interaction.

1. Introduction

For many important infectious diseases, the main tool used in the quest for management or eradication lies in vaccination against infection. Yet, while there is no doubting the private and public benefits from widespread vaccine uptake, there are important fundamental questions about vaccination markets (and vaccination demand) that remain unanswered.

Vaccinations are generally believed to be prima facie examples of actions with strong positive externalities. The idea driving this belief is beguilingly simple. If someone is vaccinated against an infectious disease, then this individual ceases to be a source or conduit of infection to other individuals.[2] Once positive external effects from vaccination are recognized, it follows naturally that if these are not properly internalized by individuals, then there is a strong case for public intervention, since a self-interested individual would tend to under-vaccinate relative to the socially optimal level.[3] This line of thinking, although intuitively appealing, turns out to depend delicately on the details of the environment, i.e. on the characteristics of the population, the disease and the available vaccine. In a controversial paper, Francis (1997) shows, using an infinite horizon susceptible-infected-removed model of epidemics without spontaneous recovery, that the market for vaccinations is efficient (i.e. it induces the socially optimal outcome) if individuals are ex ante homogeneous, the vaccine is perfect and vaccination can be taken at any point in time. The Francis welfare theorem goes against the grain of the general understanding of the properties of vaccination markets. In fact, Francis (1997) himself seems reluctant to claim any generality of the result, and indeed provides a long list of modifications to the model that could potentially alter his counter-intuitive finding.[4,5] Despite a growing literature on the

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[2] In epidemiology, the well-known concept of herd immunity, i.e. that a disease can be eradicated without vaccinating the entire population, is itself a reflection of the idea of externalities in vaccination.

[3] For an interesting instance of private individuals over-vaccinating in equilibrium relative to the social optimum, see Liu et al. (2012).

[4] Curiously, he does not pinpoint the central properties driving his result.

[5] A result related to that of Francis (1997) holds in an SI (susceptible-infected) model with non-vaccine prevention, as shown in Toxvaerd (2009).

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economics of vaccination, we believe that there are many facets of vaccination markets that are not yet fully understood. Furthermore, a better understanding of the role of externalities in decentralized vaccination decisions may also inform policy recommendations. Specifically, we will seek to answer the following two questions: (i) are decentralized vaccination markets efficient in the sense that they maximize social welfare? and (ii) if not, what is the source of the inefficiencies?

In addressing these questions, we will conduct a formal analysis by drawing heavily on methods and insights from economics. We do so for two distinct reasons. First, in answering what an "optimal" policy is, one must explicitly identify and address the basic tradeoffs that must be resolved. In practice, this means that one must not focus only on the direct costs of infection, but also on broader social costs to the economic environment and the costs involved in controlling the disease. The tools of economics are explicitly developed to analyze this kind of tradeoffs.[6] Second, when dealing with decentralized markets in which individuals make (possibly) non-coordinated decisions on whether to get vaccinated, one must explicitly model individual decision making. In doing so, it must be recognized that individuals do not merely respond passively in a privately optimal way to what other individuals do, but may also be strategically sophisticated and take into account their conjectures about other individuals' future actions. In considering such situations, economics and game theory again provide the adequate tools of analysis. There is ample evidence that individual behavior is critical for understanding aggregate disease evolution, as emphasized by Auld (2003), Reluga (2010), Fenichel et al. (2011) and Fenichel (2013).

In this paper, we analyze under what conditions the Francis welfare theorem holds. We argue that there are two central features of the Francis (1997) framework that lead equilibrium vaccination decisions to coincide with the social optimum. These are that (a) individuals are ex ante homogenous and (b) that there is no strategic interaction in vaccination decisions. Our results suggest that his welfare theorem is highly sensitive to assumptions that are made about the properties of the disease, the vaccine, and the market.

Before explaining the reasons for these results, we start by explaining why the welfare theorem is true under the sufficient conditions of Francis (1997). When there is an infinite horizon, the vaccine is perfect, there is no possible recovery and the vaccination timing is perfectly flexible, then there are no strategic interactions. This is because an individual's optimal vaccination policy (under decentralized decision making) is myopic in the sense that if the current prevalence is at or above a critical value, then it is optimal to vaccinate; otherwise, it is optimal not to vaccinate. A consequence of this is that, no matter what other individuals are currently doing or will do in the future, the individual's optimal policy is the same. That is, the individual's decision is independent of the actions of other individuals. Moreover, when individuals are ex ante identical, they have the same optimal policy. But this implies that a decentralized equilibrium has to be symmetric. Last, because of symmetry, all individuals are perfectly protected against infection at the same time (because they all vaccinate at the same time) and hence there are no external effects in equilibrium.[7] As a consequence, the equilibrium under decentralized decision making is socially efficient. Clearly, this result rests in an important way on the ex ante homogeneity of the population, as recognized by Gersovitz (2003) and Kessing and Nuscheler (2003). Ex ante heterogeneity also is central to the study by Veliov (2005), while Fenichel (2013) emphasizes that endogenous (ex post) heterogeneity may have important effects on the social optimality of equilibrium levels of social distancing (i.e. infection reducing strategies).

Given the insight we have just outlined, it is easy to see why relaxing ex ante homogeneity may invalidate the welfare theorem. With heterogeneous agents, it is again the case that an individual's optimal policy under decentralized decision making is myopic; therefore, there are no strategic interactions. However, because of heterogeneity, the different individuals' critical prevalence values differ, which means that an equilibrium is not symmetric. But asymmetry implies that there are external effects in equilibrium, because non-vaccinated individuals are influenced by the decisions of vaccinating individuals. Hence, the equilibrium in this case need not be socially efficient.

It is a priori less clear what role is played by the assumptions of an infinite planning horizon, by the impossibility of recovery or by the perfect flexibility in vaccination timing. In each of these cases, it turns out that an individual's optimal policy under decentralized decision making is not myopic. Thus in any time period, the individual's decision depends on the prevalence in that time period as well as on the expected prevalence in future time periods. But since the prevalence in future time periods depends on the actions of other individuals, strategic interactions are present. As a result, there are external effects; thus equilibrium vaccination decisions are not generally socially efficient.

We note that when there are strategic interactions, the equilibrium may or may not be ex post symmetric. This means that heterogeneity is a sufficient but not a necessary condition for decentralized equilibrium vaccine uptake to be socially inefficient. This finding also serves to emphasize that while ex ante heterogeneity is indeed an important source of inefficient vaccine uptake, the presence of strategic interactions (which we show can stem from multiple sources) represents a distinct source for possible inefficiencies, even when the population is homogeneous. This finding is important, because it shows that wrong policy conclusions may result from a single-minded focus on homogeneity/symmetry.

As we will argue in detail below, the Francis welfare theorem holds under a special (but by no means contrived) set of assumptions. Nonetheless, its importance lies in the fact that these modeling assumptions are the natural starting point of any analysis of vaccination externalities. The welfare theorem therefore serves as a benchmark for the economics of vaccination, against which one can compare less stylized and more realistic models. In particular, our analysis makes clear that it is unhelpful simply to assert that decentralized vaccination decisions involve "externalities", since the nature and causes of externalities (and therefore their possible remedies) depend in delicate ways on the chosen modeling assumptions.

1.1. Related literature

The economic control of vaccination markets was first studied formally by Hethcote and Waltman (1973) and Morton and Wickwire (1974). In these papers, and in many subsequent contributions building on this work (such as Veliov, 2008 and Hansen and Day, 2011), a central planner controls the vaccination decisions of each and every individual in the population in order to maximize some criterion of social welfare. In doing so, they not only uncovered the basic tradeoffs involved in disease control, but they also established the benchmark of optimality that any policy intervention should be measured against. These early papers did not consider individual decision making and hence did not

[6] For nice illustrations of the kind of tradeoffs that disease control involves, see e.g. Smith et al. (2009) and Keogh-Brown et al. (2010).

[7] The fact that immune individuals are unaffected by others' vaccination decisions (and indeed by any changes in disease prevalence) was first noted by Brito et al. (1991), where they used this insight to show that mandatory vaccination of the entire population would not be socially optimal.

explicitly consider the possibility of uninternalized externalities between individuals' decisions on whether to vaccinate. Fine and Clarkson (1986) seem to have made the first formal analysis of vaccination decisions featuring such considerations. Stiglitz (1988) contains a verbal discussion of the problem of externalities, and proposes government intervention in order to counter a perceived market failure.[8]

General considerations on the economics of vaccination demand and production can be found in Kremer (2000a, 2000b), and recent reviews of the economic epidemiology literature can be found in Gersovitz (2011) and in Klein et al. (2007). The first formal treatment within a mainstream economic setting is that of Brito et al. (1991). They find in a static setting with heterogeneous individuals that under decentralized decision making, individuals under-vaccinate because they do not internalize the positive benefits to other individuals.[9] The same is true in the model considered by Geoffard and Philipson (1997) which, although dynamic, disregards the dynamics of the vaccination decisions themselves (vaccination is assumed to take place at birth).[10] Boulier et al. (2007) consider a dynamic vaccination model with non-dynamic vaccination decisions and recover the inefficiency result of Brito et al. (1991). Similarly, Barrett (2003) finds market failures in a dynamic model with homogeneous individuals but with non-dynamic vaccination choices. Last, Francis (2004) also finds a similar result, but assumes that individuals do not discount the future, which renders the time profile of infections irrelevant. Gersovitz (2003) considers a vaccination model in which individuals care about their own welfare, but where the planner also cares about the welfare of future generations. He shows that if not for this difference in objectives, the planner and the representative household would agree on the optimal policy. Last, Francis (2007) studies a fully dynamic setting with a heterogeneous population and allegedly recovers the welfare theorem in this setting.

There are a number of papers that study vaccination in different settings under either centralized or decentralized decision making (but which do not compare the two). In the former category are contributions by Hethcote and Waltman (1973), Morton and Wickwire (1974), Wickwire (1975) and Behncke (2000). In the latter category are contributions by Bauch and Earn (2004), Bauch (2005), Chen (2006), Cojocaru et al. (2007), Chen and Cottrell (2009) and Goyal and Vigier (2014). Sadique et al. (2005) take a somewhat different approach and consider decision theoretical aspects of vaccination.

Last, there is a related but distinct literature on non-vaccine protective measures (such as condoms and bed nets). See e.g., Toxvaerd (2009) for a review of that literature.[11]

In recent years, a new breed of papers has emerged, dealing with different aspects of "behavioral epidemiology", i.e. models in which agents either behave in a boundedly rational manner, or in which information about diseases or the value of vaccination spreads gradually via word-of-mouth learning (see e.g. Medlock et al., 2009; d'Onofrio et al., 2013; Bauch et al., 2013; Fenichel and Wang, 2013). These types of approaches seem like the natural next step once the more stylized models of economic epidemiology have been well understood.

For completeness, we should mention three types of vaccination policies that are staples of the mathematical epidemiology literature. First, many papers consider so-called pulse vaccination. Pulse vaccination is a discrete increase in the vaccination rate at discrete points in time (e.g., the vaccination of some subpopulation at regular intervals). Second, many papers consider vaccination of all susceptibles in the population at a constant rate. Last, many papers consider the vaccination of some constant fraction of all newborns. Common to these policies is that they are not derived as part of an explicit optimization programme or as an equilibrium outcome. While they are very useful in understanding how vaccinations change the mechanics of disease propagation, they are not useful for making policy recommendations unless coupled with an explicit welfare criterion like that found in the economic epidemiology literature.

The paper is structured as follows. In Section 2, we set out the model. In Section 3, we derive a discrete-time analog of the welfare theorem in Francis (1997). We then show that the welfare results may cease to hold if individuals may recover spontaneously from the disease (Section 4), if vaccines are imperfect (Section 5), if individuals are ex ante heterogeneous (Section 6), if the timing of vaccination is inflexible (Section 7) or if the planning horizon is finite (Section 8). Section 9 concludes while the Appendix contains further details on the analysis.

In each section, we note which existing papers share properties with our particular version of the model. The present paper therefore also serves to tie together seemingly disparate models and results in the literature.

2. The basic model

The vaccination model we consider is based on the classical *susceptible–infected–recovered* (*SIR*) model. In this model, individuals go through the health states indicated in the name. In the classical (uncontrolled) SIR model, susceptible individuals in a population mix with other individuals and are thereby exposed to an infectious disease. Once infected, an individual transitions from the susceptible to the infected state. Upon infection, an individual may recover spontaneously from the infection, thereby making a transition to the recovered (or removed) state. Once recovered, the individual is no longer susceptible. Vaccination models are typically SIR models in which vaccination takes an individual directly from the susceptible state to the recovered state (thereby bypassing the infected state) if the vaccine is perfect and permanent, and to a temporary protected state otherwise.

For simplicity and convenience, we consider a population with three individuals (individuals 1, 2, and 3). There is an infectious disease circulating in the population, and each individual can be in one of three health states in any given period: *susceptible*, *infected*, or *protected*. An individual can become protected by either (i) getting vaccinated; or (ii) acquiring natural immunity after recovering from an infection. The per period cost of being infected (in utils) is $\pi > 0$. Assume that a vaccine exists and that a susceptible individual i, $i = 1, 2, 3$, can get vaccinated in any period at a cost $c_i > 0$. The vaccine confers lifelong immunity (i.e. it does not wane). Individuals discount the future using the discount factor $\delta \in [0, 1)$.

The transmission process of the disease is given as follows. If a susceptible individual chooses not to be vaccinated in some period, then that individual becomes infected in the next period with a probability that is proportional to the fraction of other individuals that are currently infected. This proportionality factor is the transmission probability $\beta \in (0, 1]$. If the individual chooses

[8] Stiglitz' discussion has initiated response and counterresponses by Philipson (2000), Gersovitz (2003), Gersovitz and Hammer (2003) and Gersovitz (2011), respectively.

[9] The analysis of Kureishi (2009) builds on that of Brito et al. (1991) and therefore arrive at similar conclusions. Other related (static) analyses include Xu (1999), Heal and Kunreuther (2005) and Althouse et al. (2010).

[10] While the analysis of Geoffard and Philipson (1997) is mainly positive, it contains some discussion of normative issues.

[11] We note that there exists a smaller literature on the supply side of the market for vaccinations, i.e., the industrial organization of vaccine production. Kremer (2000a, 2000b), Kessing and Nuscheler (2003), Kremer and Snyder (2003), Forslid (2006) and Forslid and Herzing (2008) study different settings with a monopolist producer. Mechoulan (2005), Kremer et al. (2008) and Boulier (2009) analyze different aspects of the regulation of monopolists and also study aspects of imperfectly and perfectly competitive markets for vaccines.

to be vaccinated, then the transmission probability decreases to $\beta(1-\varepsilon)$, where $\varepsilon \in [0,1]$ is the efficacy of the vaccine. For example, if in period t individual 3 is the only infected individual, then the probability that individual 1 will be infected in period $t+1$ is $\beta/2$ if individual 1 chooses not to be vaccinated in period t, and it drops to $\beta(1-\varepsilon)/2$ if individual 1 is vaccinated in period t. For simplicity and without loss of generality, set $\beta=1$. Assume that initially (i.e., at $t=0$) individual 3 is infected and the other two individuals (individuals 1 and 2) are susceptible.

If an individual is infected in period t, then the individual recovers with probability $\rho \in [0,1]$ in period $t+1$ for any t. We assume that a recovered individual is perfectly immune to the disease and cannot be reinfected. To facilitate our discussion, we will assume that if the vaccine is not perfectly effective (i.e., if $\varepsilon<1$), then the probability of recovery ρ is 0. This is done purely for ease of notation: if infected individuals can recover and become fully immune when the vaccine efficacy is less than 100%, then we would have to keep track of two types of protected individuals (those protected imperfectly by vaccines, and those protected perfectly through having been infected) instead of just one type.

In any period, the state of the population is given by the two-tuple (I,P), where I is the number of infected individuals and P is the number of individuals that are protected in the current period. With three individuals, the state space is thus

$$\Omega = \{(3,0),(2,0),(1,0),(0,0),(2,1),(1,1),(0,1),(1,2),(0,2),(0,3)\}.$$

Individuals make decisions regarding whether to vaccinate or not only when they are susceptible, and at least one susceptible individual exists in the subset $\Omega_S \subset \Omega$, where

$$\Omega_S = \{(2,0),(1,0),(0,0),(1,1),(0,1),(0,2)\}.$$

For convenience, define $\Omega_S^i \subset \Omega_S$ to be the set of population states in which individual i is susceptible. Similarly, let $\Omega_P^i \subset \Omega$ be the set of population states in which individual i is protected.

We have chosen to conduct the analysis in this simple three-individual setting for two reasons. First, it allows us to make a number of points using elementary mathematics, thereby making the results easily accessible to those mainly interested in the policy relevant conclusions. Second, the setting allows us to conduct the analysis analytically and to obtain closed form solutions throughout.

Throughout, we assume that individuals non-cooperatively minimize their individual expected, discounted lifetime cost, while the social planner minimizes the sum of all individuals' expected, discounted lifetime cost. When analyzing decentralized decision-making, we restrict attention to Markov strategies where a susceptible individual's action in any period t depends only on the state of the population in that period and on calendar time t (see e.g. Maskin and Tirole, 2001 for more details). More formally, a pure strategy σ_i for individual i, $i=1,2$, specifies for any t whether to get vaccinated or not for every element in Ω_S^i, i.e., $\sigma_i : \Omega_S^i \times \mathbb{N} \to \{0,1\}$, where $\sigma_i=1$ denotes *vaccinate* and $\sigma_i=0$ denotes *not vaccinate*.[12] In a Markov perfect equilibrium, every individual's strategy is optimal given the strategies of the other individuals.

The social planner, on the other hand, chooses for every state in Ω_S which susceptible individual or individuals to vaccinate in order to minimize social cost, i.e., the sum of all individuals' expected, discounted lifetime cost. Note that a susceptible individual has probability 0 of becoming infected in states $(0,0)$, $(0,1)$, or $(0,2)$. Therefore, in any of these states, a susceptible individual would not vaccinate in equilibrium, and the social planner would not choose to vaccinate any susceptible individual. Throughout, we assume that the population under consideration is closed, i.e., there are no births or deaths. We do so in order to focus exclusively on the effects that decentralization of vaccination decisions has on social welfare.[13]

We use dynamic programming to solve each susceptible individual's optimization problem under decentralized decision-making, as well as the social planner's optimization problem.[14] For the decentralized case, let $U_{i,t}(\omega;\sigma_j)$, $i=1,2$, $i\neq j$, denote the value of individual i's cost minimization problem in period t—i.e., the minimum expected, discounted lifetime cost for individual i starting in period t—if the state of the population in t is $\omega\in\Omega_S^i$, individual i is susceptible in t, and individual j uses the strategy σ_j. Similarly, let $V_{i,t}(\omega;\sigma_j)$ denote individual i's expected, discounted lifetime cost starting in period t if individual i is protected by vaccination in t, the state of the population in t is $\omega\in\Omega_P^i$, and individual j uses the strategy σ_j. If individual i is infected in period t, let $\underline{U}_{i,t}$ denote the individual's expected, discounted lifetime cost starting in t. Note that, given our assumption on the recovery process, $\underline{U}_{i,t}$ satisfies the following condition:

$$\underline{U}_{i,t} = \pi + \delta(1-\rho)\underline{U}_{i,t+1}. \tag{1}$$

To see what individual i should do in period t when she is susceptible, the population is in state $(I,P)\in\Omega_S^i$, and individual j is using strategy σ_j, consider individual i's options.

- By vaccinating, individual i incurs the cost c_i in the current period. In the following period, individual i becomes protected with probability $1-(1-\varepsilon)I/2$ and infected with probability $(1-\varepsilon)I/2$. If she is protected by vaccination in the following period when the state of the population is ω', then her expected, discounted lifetime cost from that point onwards is $V_{i,t+1}(\omega';\sigma_j)$. If she is infected in the following period, then her expected, discounted lifetime cost from that point onwards is $\underline{U}_{i,t+1}$. Therefore, the discounted, expected lifetime cost from vaccinating in the current period is

$$\Gamma^V_{i,t}((I,P)) \equiv c_i + \delta\left[(1-\varepsilon)\frac{I}{2}\underline{U}_{i,t+1} + \left(1-(1-\varepsilon)\frac{I}{2}\right)\times \sum_{\omega'\in\Omega_P^i} Q_V(\omega'|(I,P);\sigma_j)V_{i,t+1}(\omega';\sigma_j)\right], \tag{2}$$

  where $Q_V(\omega'|(I,P);\sigma_j)$ is the probability that—conditional on individual i not getting infected—the state of the population in the next period is ω' given that the current state is (I,P), individual i vaccinates in the current period, and individual j uses strategy σ_j.
- By not vaccinating, individual i incurs no cost in the current period. In the following period, individual i remains susceptible with probability $1-(I/2)$ and becomes infected with probability $I/2$. If she remains susceptible in the following period when the state of the population is ω', then her expected, discounted

[12] Since, by assumption, individual 3 will never be susceptible, there is no need to consider individual 3's strategy.

[13] Gersovitz (2003) shows that if the population grows over time, then the planner's solution will not coincide with the decentralized equilibrium outcome. This is because the individuals only consider themselves, while the planner also considers the welfare of future generations. The right interpretation of this result is not that decentralization per se leads to suboptimal vaccination uptake. What drives the difference between the optimal outcome and the equilibrium outcome is that the two decision makers are looking at the welfare of *two different groups* (the planner values current *and* future generations, while the household only cares about the present generation). Thus the different outcomes are hard-wired into the preferences. This does not mean that the distortion in vaccination decisions brought about by a growing population is not important. It clearly is, but it is different in nature from the ones studied here. We conjecture, but have not shown, that if individuals have dynastic preferences (i.e., they care about the welfare of their own offspring), then the welfare theorem is restored even with a growing population.

[14] For a good exposition of dynamic programming techniques, see e.g. Stokey et al. (1989).

lifetime cost from that point onwards is $U_{i,t+1}(\omega';\sigma_j)$. If she is infected in the following period, then her expected, discounted lifetime cost from that point onwards is $\underline{U}_{i,t+1}$. Therefore, the expected, discounted lifetime cost from not vaccinating in the current period is

$$\Gamma^{NV}_{i,t}((I,P)) \equiv \delta\left[\frac{I}{2}\underline{U}_{i,t+1} + \left(1-\frac{I}{2}\right)\times \sum_{\omega' \in \Omega^i_S} Q_{NV}(\omega'|(I,P);\sigma_j)U_{i,t+1}(\omega';\sigma_j)\right], \tag{3}$$

where $Q_{NV}(\omega'|(I,P);\sigma_j)$ is the probability that—conditional on individual i not getting infected—the state of the population in the next period is ω' given that the current state is (I,P), individual i does not vaccinate in the current period, and individual j uses strategy σ_j.

The above observations thus imply that the value function $U_{i,t}$ must satisfy the Bellman equation

$$U_{i,t}((I,P);\sigma_j) = \min\{\Gamma^V_{i,t}((I,P)), \Gamma^{NV}_{i,t}((I,P))\}. \tag{4}$$

To derive the value function $V_{i,t}$, note that if individual i is protected by vaccination in period t, the state of the population is (I,P), and individual j uses the strategy σ_j, then with probability $1-(1-\varepsilon)I/2$ individual i will remain protected in the following period; from that point onwards, individual i's expected, discounted lifetime cost is $V_{i,t+1}(\omega';\sigma_j)$ if ω' is the state of the population in the next period. With probability $(1-\varepsilon)I/2$, individual i will become infected in the next period, and her expected, discounted lifetime cost from then on is $\underline{U}_{i,t+1}$. Therefore, $V_{i,t}$ solves

$$V_{i,t}((I,P);\sigma_j) = \delta\left[(1-\varepsilon)\frac{I}{2}\underline{U}_{i,t+1} + \left(1-(1-\varepsilon)\frac{I}{2}\right)\times \sum_{\omega' \in \Omega^i_P} Q_V(\omega'|(I,P);\sigma_j)V_{i,t+1}(\omega';\sigma_j)\right]. \tag{5}$$

An equilibrium in the decentralized case is defined formally below.

Definition 1. If, for $i,j = 1,2$ and all $(I,P) \in \Omega^i_S$,

$$\sigma_i((I,P),t) = \begin{cases} 1 & \text{if } \Gamma^V_{i,t}((I,P)) \geq \Gamma^{NV}_{i,t}((I,P)) \\ 0 & \text{if } \Gamma^V_{i,t}((I,P)) \leq \Gamma^{NV}_{i,t}((I,P)) \end{cases},$$

then (σ_1,σ_2) is an equilibrium of the decentralized market.

Now, let us consider how to solve the social planner's problem. For ease of exposition, in this section we will assume that all individuals have the same vaccination cost when we formally state the social planner's optimization problem. In terms of problem set-up, nothing is substantively altered when individuals' vaccination costs differ, and we defer the analysis of this case to Section 6. To proceed, define $W_t(\omega)$ to be the value of the planner's problem—i.e., the minimum sum of all individuals' expected, discounted lifetime cost—if the current state of the population is $\omega \in \Omega$. Now, if $\omega \in \Omega_S$, then the planner has to choose how many susceptible individuals to vaccinate in the current period. If the planner chooses to vaccinate n susceptible individuals, then—assuming all individuals have the same cost of vaccination c—a total cost of nc is incurred in the current period; and if the state of the population changes to ω' in the following period, then the minimum sum of all expected, discounted lifetime cost from then on is $W_{t+1}(\omega')$. Therefore, if $(I,P) \in \Omega_S$, $W_t((I,P))$ must solve the Bellman equation

$$W_t((I,P)) = I\pi + \min_n\left\{nc + \delta \sum_{\omega' \in \Omega} Q(\omega'|(I,P),n)W_{t+1}(\omega')\right\}, \tag{6}$$

where $Q(\omega'|(I,P),n)$ denotes the probability that the state of the population in the next period is ω' given that the current state is (I,P) and n susceptible individuals are vaccinated in the current period. For $(I,P) \notin \Omega_S$, $W_t((I,P))$ satisfies

$$W_t((I,P)) = I\pi + \delta \sum_{\omega' \in \Omega} Q(\omega'|(I,P),0)W_{t+1}(\omega'). \tag{7}$$

For later use, note that a policy that minimizes social cost (i.e., solves the planner's cost minimization problem) is said to be "efficient" or "first-best optimal".

A list of the notation used here, as well as their descriptions, is given in the table below.

Notation	Description
π	cost of being infected
c_i	cost of vaccination for individual i
δ	discount factor
β	transmission probability
ε	vaccine efficacy
ρ	probability of recovery from infection
I	number of infected individuals
P	number of protected individuals
Ω	set of all population states (I,P)
Ω_S	set of population states with at least one susceptible individual
Ω^i_S	set of population states in which individual i is susceptible
Ω^i_P	set of population states in which individual i is protected
σ_i	vaccination strategy of individual i
$U_{i,t}$	individual i's value function in period t given that individual i is susceptible
$V_{i,t}$	individual i's value function in period t given that individual i is protected
$\underline{U}_{i,t}$	individual i's value function in period t given that individual i is infected
Q	transition probability function for the social planner's problem
Q_V	transition probability function for an individual's decision problem if the individual is vaccinated
Q_{NV}	transition probability function for an individual's decision problem if the individual does not vaccinate
W_t	the social planner's value function in period t

3. The welfare theorem

In this section, we present a discrete-time version of the Francis welfare theorem. The central assumptions are as follows.

A1 Individuals cannot recover from infection.
A2 Vaccination confers perfect immunity.
A3 Individuals are ex ante homogeneous.
A4 Individuals can vaccinate in any time period.
A5 Individuals are infinitely-lived.

With these assumptions in place, we can now state the following result.

Welfare theorem *The equilibrium pattern of vaccination is socially optimal if assumptions A1–A5 are satisfied.*

In this section, we prove the Francis welfare theorem in our setting. In subsequent sections, we relax each of Assumptions

A1–A5 in turn and study the implications for the social optimality of equilibrium vaccination patterns.

3.1. Equilibrium vaccination

Following the stated assumptions, set $\rho=0$, $\varepsilon=1$, and $c_1=c_2=c$.[15] Note that, in this case, an individual is protected if and only if the individual has been vaccinated. Let us consider individual i's decision problem. Assuming that individuals are infinitely-lived, we can omit the time variable t as a state variable of the optimization problem; hence, we have, for all t, that

$$U_{i,t}(\omega;\sigma_j)=U_i(\omega;\sigma_j), \quad \forall\omega\in\Omega_S^i, \quad \forall\sigma_j, \tag{8}$$

$$V_{i,t}(\omega;\sigma_j)=V_i(\omega;\sigma_j), \quad \forall\omega\in\Omega_P^i, \quad \forall\sigma_j, \tag{9}$$

and

$$\underline{U}_{i,t}=\underline{U}_i; \tag{10}$$

in addition,

$$\sigma_i(\omega,t)=\sigma_i(\omega), \quad \forall\omega\in\Omega_S^i. \tag{11}$$

Since $\varepsilon=1$, (5) implies that $V_i(\omega;\sigma_j)=0$ for all $\omega\in\Omega_P^i$ and all σ_j. From (1), we have $\underline{U}_i=\pi/(1-\delta)$ when $\rho=0$.

Given the stated assumptions, the transition probability function $Q_{NV}(\omega'|(I,P);\sigma_j)$ can be given by the transition matrix below:

		(2, 0)	(1, 0)	(1, 1)
(1, 0)	if $\sigma_j=0$	$\frac{1}{2}$	$\frac{1}{2}$	0
	if $\sigma_j=1$	0	0	1
(1, 1)		0	0	1

The rows of the matrix correspond to the current state of the population, while the columns indicate the state of the population in the next period. The number in row a and column b gives the probability that the state of the population will move from the state represented by row a to the state represented by column b, conditional on individual i staying susceptible in the next period. As an example, in the above matrix, the entry in row 1 and column 3 tells us that the probability that the population will move from state $(1,0)\in\Omega_S^i$ to state $(1,1)\in\Omega_S^i$ is 0 if $\sigma_j((1,0))=0$, and that this probability is 1 if $\sigma_j((1,0))=1$. For brevity, only the elements in Ω_S^i that are needed for solving individual i's optimization problem are shown. Note that since $V_i(\omega;\sigma_j)=0$ for all $\omega\in\Omega_P^i$ and all σ_j, it is not necessary to specify the transition function $Q_V(\omega'|(I,P);\sigma_j)$ when solving a susceptible individual's problem.

Using the general Bellman equation (4) and the conditions stated above, it can be shown that

$$U_i((2,0);\sigma_j)=\min\left\{c,\frac{\delta\pi}{1-\delta}\right\},$$

$$U_i((1,0);\sigma_j) = \begin{cases} \min\left\{c,\delta\left[\frac{1}{2}\left(\frac{\pi}{1-\delta}\right)+\frac{1}{2}U_i((1,1);\sigma_j)\right]\right\} & \text{if } \sigma_j((1,0))=1 \\ \min\left\{c,\delta\left[\frac{1}{2}\left(\frac{\pi}{1-\delta}\right)+\frac{1}{2}\left(\frac{1}{2}U_i((1,0);\sigma_j)+\frac{1}{2}U_i((2,0);\sigma_j)\right)\right]\right\} & \text{if } \sigma_j((1,0))=0 \end{cases},$$

and

$$U_i((1,1);\sigma_j)=\min\left\{c,\frac{\delta\pi}{(1-\delta)(2-\delta)}\right\}.$$

There are three cases to consider.

Case 1: $c>\delta\pi/(1-\delta)$.

In this case, $U_i((2,0);\sigma_j)=\delta\pi/(1-\delta)$, $U_i((1,1);\sigma_j)=\delta\pi/(1-\delta)(2-\delta)$, and

$$U_i((1,0);\sigma_j)=\begin{cases}\dfrac{\delta\pi}{(1-\delta)(2-\delta)} & \text{if } \sigma_j((1,0))=1\\ \dfrac{\delta\pi(2+\delta)}{(1-\delta)(4-\delta)} & \text{if } \sigma_j((1,0))=0\end{cases}.$$

In equilibrium, no susceptible individual ever chooses to vaccinate.

Case 2: $\delta\pi/(1-\delta)(2-\delta)<c\le\delta\pi/(1-\delta)$.

In this case, $U_i((2,0);\sigma_j)=c$, $U_i((1,1);\sigma_j)=\delta\pi/(1-\delta)(2-\delta)$, and

$$U_i((1,0);\sigma_j)=\begin{cases}\dfrac{\delta\pi}{(1-\delta)(2-\delta)} & \text{if } \sigma_j((1,0))=1\\ \dfrac{\delta(c(1-\delta)+2\pi)}{(1-\delta)(4-\delta)} & \text{if } \sigma_j((1,0))=0\end{cases}.$$

In equilibrium, a susceptible individual chooses to vaccinate in state $(2,0)$, and does not vaccinate in other states.

Case 3: $c\le\delta\pi/(1-\delta)(2-\delta)$.

In this case, $U_i((2,0);\sigma_j)=U_i((1,1);\sigma_j)=c$, and $U_i((1,0);\sigma_j)=c$ if $\sigma_j((1,0))=1$ or $\sigma_j((1,0))=0$. In equilibrium, a susceptible individual chooses to vaccinate in states $(2,0)$, $(1,1)$ and $(1,0)$ and does not vaccinate in other states.

Note that, in all three cases, a susceptible individual's vaccination decision in state $(1,0)$ does not depend on the other susceptible individual's action, i.e., a susceptible individual would choose the same action no matter what the other susceptible individual chooses. Therefore, there are no strategic interactions in this setting. Also, note that, regardless of the parameter values, a susceptible individual employs a threshold policy whereby she vaccinates if the number of individuals currently infected is above a critical number and does not vaccinate otherwise.

3.2. Socially optimal vaccination

Given the assumptions that individuals are infinitely-lived and $c_1=c_2=c$, we have, for all t,

$$W_t(\omega)=W(\omega), \quad \forall\omega\in\Omega. \tag{12}$$

In addition, the transition probability function Q is given by the transition matrix below (not all of the states in Ω are shown, since not all of them are needed for solving the social planner's optimization problem):

		(3, 0)	(2, 0)	(2, 1)	(1, 0)	(1, 1)	(1, 2)
(3, 0)		1	0	0	0	0	0
(2, 0)	if $n=0$	1	0	0	0	0	0
	if $n=1$	0	0	1	0	0	0
(2, 1)		0	0	1	0	0	0
(1, 0)	if $n=0$	$\frac{1}{4}$	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0
	if $n=1$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0
	if $n=2$	0	0	0	0	0	1
(1, 1)	if $n=0$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0
	if $n=1$	0	0	0	0	0	1
(1, 2)		0	0	0	0	0	1

From (6), (7), and the stated assumptions, we have $W((3,0))=3\pi/(1-\delta)$, $W((2,1))=2\pi/(1-\delta)$, and $W((1,2))=\pi/(1-\delta)$. Again, there are three cases to consider.

Case 1: $c>\delta\pi/(1-\delta)$.

In this case, $W((2,0))=\pi(2+\delta)/(1-\delta)$, $W((1,1))=2\pi/(1-\delta)(2-\delta)$, and $W((1,0))=\pi(2\delta^2+3\delta+4)/(1-\delta)(4-\delta)$. In the social optimum, no susceptible individual is vaccinated.

Case 2: $\delta\pi/(1-\delta)(2-\delta)<c\le\delta\pi/(1-\delta)$.

In this case, $W((2,0))=c+2\pi/(1-\delta)$, $W((1,1))=2\pi/(1-\delta)(2-\delta)$, and $W((1,0))=4\pi-2c\delta^2+2c\delta+3\delta\pi/(1-\delta)(4-\delta)$. In the social optimum, vaccination occurs only in state $(2,0)$.

[15] We will henceforth assume that when individuals are homogeneous, all individuals' vaccination cost is c.

Case 3: $c \le \delta\pi/(1-\delta)(2-\delta)$.

In this case, $W((2,0)) = c + 2\pi/(1-\delta)$, $W((1,1)) = c + \pi/(1-\delta)$, and $W((1,0)) = 2c + \pi/(1-\delta)$. In the social optimum, all susceptible individuals are vaccinated in states $(2,0)$, $(1,1)$, and $(1,0)$.

A comparison of the social optimum and the equilibrium shows that the timing and extent of vaccination in equilibrium is socially optimal.

4. The model with spontaneous recovery

We now relax Assumption A1 and consider the possibility that infected individuals may recover spontaneously from the disease. This is the case studied by Francis (2004) without discounting and by Francis (2007) with discounting. Reluga and Galvani (2011) consider both recovery and waning of immunity. Gersovitz (2003) also considers recovery, but from the infected to the susceptible state, so his results are therefore not directly comparable to ours.[16] Whether spontaneous recovery is possible is not merely a modeling choice, but is determined by the disease at hand.

When recovery from infection is possible, it is the case that there are strategic interdependencies between individuals' decisions. These create the potential for inefficiencies, as we will now show.

Turning to our setup, assume that $\rho > 0$ (while keeping all the other assumptions). In this case, there are two ways for an individual to become protected: by vaccinating while the individual is susceptible, or by recovering after being infected.

Proposition 2. *The equilibrium pattern of vaccination may be socially suboptimal if individuals may recover spontaneously and become immune.*

4.1. Equilibrium vaccination

Consider individual i's decision problem. Since individuals are infinitely-lived, conditions (8)–(11) hold, with $V_i(\omega; \sigma_j) = 0$ for all $\omega \in \Omega_P^i$ and all σ_j. In addition, following from (1), the value of being infected for individual i is $\underline{U}_i = \pi/(1-\delta(1-\rho))$. The transition probability function $Q_{NV}(\omega' | (I, P); \sigma_j)$ is given by the transition matrix

$$\begin{array}{ll|c|c|c|c|c|} & & (2,0) & (1,0) & (1,1) & (0,1) & (0,2) \\ \hline (1,0) & \text{if } \sigma_j = 0 & \frac{1}{2}(1-\rho) & \frac{1}{2}(1-\rho) & \frac{1}{2}\rho & \frac{1}{2}\rho & 0 \\ & \text{if } \sigma_j = 1 & 0 & 0 & 1-\rho & 0 & \rho \\ \hline (1,1) & & 0 & 0 & 1-\rho & 0 & \rho \\ \hline \end{array}.$$

Since a susceptible individual has probability 0 of ever getting infected in states $(0,2)$ and $(0,1)$, $U_i((0,2); \sigma_j) = U_i((0,1); \sigma_j) = 0$, $\forall \sigma_j$. Also, $U_i((2,0); \sigma_j) = \min\{c, \delta\pi/(1-\delta(1-\rho))\}$.

4.2. Socially optimal vaccination

With infinitely-lived individuals, (12) holds. The transition probability function Q is given in Appendix A. It can be shown that $W((0,3)) = W((0,2)) = W((0,1)) = 0$, $W((1,2)) = \pi/(1-\delta(1-\rho))$, $W((2,1)) = 2\pi/(1-\delta(1-\rho))$, and $W((3,0)) = 3\pi/(1-\delta(1-\rho))$. The following example shows that the equilibrium need not coincide with the solution to the planner's problem (for more details of the example, see Appendix A).

4.3. Inefficiency of equilibrium: Example 1

Suppose that $\delta\pi/(1-\delta(1-\rho)) > c > \delta\pi/(1-\delta(1-\rho))(2-\delta(1-\rho))$. These parameter restrictions imply that, in equilibrium, a susceptible individual vaccinates in state $(2,0)$ and does not vaccinate in state $(1,1)$. Furthermore, in state $(1,0)$, both susceptible individuals choose not to vaccinate if $c \ge \underline{c}$, where

$$\underline{c} \equiv \frac{\delta\pi(4-2\delta+3\delta\rho)}{2(1-\delta(1-\rho))(2-\delta(1-\rho))^2} \in \left(\frac{\delta\pi}{(1-\delta(1-\rho))(2-\delta(1-\rho))}, \frac{\delta\pi}{(1-\delta(1-\rho))} \right).$$

For the social planner's problem, given the stated parameter restrictions, it is optimal to vaccinate the only susceptible individual in state $(2,0)$, and it is optimal to not vaccinate the only susceptible individual in state $(1,1)$. In state $(1,0)$, it is optimal to vaccinate no one if $c \ge \overline{c}$, while it is optimal to vaccinate only one susceptible individual if $c \le \overline{c}$, where

$$\overline{c} \equiv \frac{\delta\pi(4-3\delta+5\delta\rho)}{(1-\delta(1-\rho))(2-\delta(1-\rho))(4-3\delta(1-\rho))}.$$

It can be verified that $\overline{c} > \underline{c}$. This means that if $c \in (\underline{c}, \overline{c})$, then in equilibrium no one vaccinates in state $(1,0)$, but the socially optimal outcome is to vaccinate one susceptible individual.

5. The model with imperfect vaccines

We now relax Assumption A2, i.e., that once vaccinated, an individual is no longer susceptible to infection. Imperfect vaccines are considered in equilibrium settings by Heal and Kunreuther (2005), Chen (2006), Chen and Cottrell (2009), Boulier (2009) and Reluga and Galvani (2011). It is also considered by Sadique et al. (2005). We model imperfect vaccines by assuming that once vaccinated, the individual still has a positive (but reduced) probability of becoming infected.[17] The decision to model vaccines as perfect is often made for simplicity, but it is ultimately an empirical question whether it is an appropriate modeling assumption for a given disease/vaccine combination.

Previewing the results of this section, recall that one consequence of perfect vaccines was that in the symmetric equilibrium, vaccinated individuals did not benefit from the vaccination of others. In other words, in equilibrium there were no external effects. With imperfect vaccines, this conclusion no longer holds. Specifically, even when vaccinated, an individual is only partially protected against infection. This means that even protected individuals benefit from the vaccination of others, since they are still potential sources of infection. Therefore, even in a fully symmetric equilibrium, there are positive externalities that are not internalized by the individuals; this drives a wedge between the equilibrium outcome and the first-best vaccination pattern.

We now turn to the analysis of the model with imperfect vaccines, i.e., $\varepsilon < 1$. We maintain all other assumptions, so that the only way for an individual to be protected is through vaccination.

Proposition 3. *The equilibrium pattern of vaccination may be socially suboptimal if vaccines are imperfect.*

5.1. Equilibrium vaccination

Given infinitely-lived individuals, conditions (8)–(11) hold. Looking at individual i's optimization problem, the value of being

[16] See footnote 13.

[17] An alternative would be to assume that vaccination is successful only with some probability and that with the remaining probability, the vaccinated individual remains susceptible. In this alternative setting, if the success or failure of the vaccine is unobservable, then a vaccinated but uninfected individual would always remain uncertain about his immunity. Furthermore, there would be scope for repeated vaccination in order to increase the probability of obtaining immunity. While this scenario is an interesting one, we focus on the simpler setting in which the vaccine is always successful but only confers limited immunity.

infected in this case is $\underline{U}_i = \pi/(1-\delta)$. The transition probabilities $Q_V(\omega'|(I,P);\sigma_j)$ given $(I,P) \in \Omega_S^i$ are as follows:

$$\begin{array}{ll|c|c|c|c|} & & (2,1) & (1,0) & (1,1) & (1,2) \\ \hline & (2,0) & 1 & 0 & 0 & 0 \\ \hline (1,0) & \text{if } \sigma_j = 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ & \text{if } \sigma_j = 1 & \frac{1}{2}(1-\varepsilon) & 0 & 0 & 1-\frac{1}{2}(1-\varepsilon) \\ \hline & (1,1) & \frac{1}{2}(1-\varepsilon) & 0 & 0 & 1-\frac{1}{2}(1-\varepsilon) \\ \hline \end{array}. \tag{13}$$

For $Q_V(\omega'|(I,P);\sigma_j)$ given $(I,P) \in \Omega_P^i$, we have the transition matrix

$$\begin{array}{ll|c|c|c|} & & (2,1) & (1,1) & (1,2) \\ \hline & (2,1) & 1 & 0 & 0 \\ \hline (1,1) & \text{if } \sigma_j = 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ & \text{if } \sigma_j = 1 & \frac{1}{2}(1-\varepsilon) & 0 & 1-\frac{1}{2}(1-\varepsilon) \\ \hline & (1,2) & \frac{1}{2}(1-\varepsilon) & 0 & 1-\frac{1}{2}(1-\varepsilon) \\ \hline \end{array}. \tag{14}$$

5.2. Socially optimal vaccination

As before, (12) holds since individuals are infinitely-lived. The matrix of transition probabilities $Q(\omega'|\omega, n)$ is given in Appendix B. The following example shows that the equilibrium need not coincide with the solution to the planner's problem.

5.3. Inefficiency of equilibrium: Example 2

Suppose that $\delta = \frac{9}{10}$, $\varepsilon = \frac{2}{3}$, and $\pi = 1$. It can be shown, using (4), (5), and the transition matrices (13) and (14), that for $c > \frac{471}{310}$, no one chooses to get vaccinated in equilibrium (for more details of this example, see Appendix B). For the planner's problem, it can be verified, using (6) and (7), that, for $c \in (\frac{119}{62}, \frac{109}{50})$, the solution is to vaccinate the only susceptible individual when there is one susceptible individual and one protected individual, and not to vaccinate anybody in other states. Since $\frac{471}{310} < \frac{119}{62}$, the solution to the planner's problem differs from the equilibrium for $c \in (\frac{119}{62}, \frac{109}{50})$.

6. The model with heterogeneous individuals

We now relax Assumption A3 and introduce ex ante heterogeneity to the model. Heterogeneity is also considered by Brito et al. (1991), Xu (1999) and Kureishi (2009), but they do so using static frameworks. Heterogeneity is also a key feature of the dynamic model of Gersovitz (2003), but takes a different form than the one considered here. Last, Francis (1997) also considers heterogeneity in a dynamic framework and finds that his welfare theorem continues to hold. His result is contradicted by ours. We should point out that heterogeneity, as introduced in this section, is ex ante heterogeneity, i.e., heterogeneity that characterizes the individuals at the outset. Ex ante homogeneity is usually imposed for ease of analysis, but in practice, individuals are heterogeneous along a number of dimensions such as susceptibility, risk attitudes and income.

To briefly preview the results of this section, recall that the Francis result held in part because in equilibrium, both the planner and the individuals chose to vaccinate all remaining susceptibles at the same time. As a consequence, in equilibrium there were no external effects, since all potential beneficiaries of a given individual's vaccination were themselves fully protected via their own vaccination. When individuals are ex ante heterogeneous, they face different trade-offs and will thus choose different patterns of vaccination (specifically, they may choose to vaccinate at different critical levels of disease prevalence). But this means that in equilibrium, there are (non-internalized) external effects. The individuals ignore these but the planner does not, leading to equilibrium under-vaccination relative to the first-best.

Turning to the analysis, suppose $c_2 \gg c_1$, where c_2 is prohibitively high so that individual 2 would never choose to get vaccinated and the social planner would not choose to vaccinate this individual, regardless of disease prevalence. We now show that the equilibrium outcome may differ from the efficient outcome.

Proposition 4. *The equilibrium pattern of vaccination may be socially suboptimal if individuals are ex ante heterogeneous.*

6.1. Equilibrium vaccination

Since individuals are infinitely-lived, conditions (8)–(11) hold, with $\underline{U}_1 = \pi/(1-\delta)$ and $V_1(\omega;\sigma_j) = 0$ for all $\omega \in \Omega_P^1$. Note that $\sigma_2(\omega) = 0$ for all $\omega \in \Omega_S^2$ given our assumption on c_2. The value $U_1((2,0);\sigma_2)$ solves $U_1((2,0);\sigma_2) = \min\{c_1, \delta\pi/(1-\delta)\}$. Let us suppose that $c_1 \le \delta\pi/(1-\delta)$ so that individual 1 would choose to get vaccinated if the other two individuals are both infected.

Now, let us consider individual 1's problem when only individual 3 is infected. With our assumptions, the value $U_1((1,0);\sigma_2)$ solves

$$\begin{aligned} U_1((1,0);\sigma_2) &= \min\left\{c_1, \delta\left[\frac{1}{2}\left(\frac{\pi}{1-\delta}\right) + \frac{1}{4}U_1((1,0);\sigma_2) + \frac{1}{4}U_1((2,0);\sigma_2)\right]\right\} \\ &= \min\left\{c_1, \delta\left[\frac{1}{2}\left(\frac{\pi}{1-\delta}\right) + \frac{1}{4}U_1((1,0);\sigma_2) + \frac{1}{4}c_1\right]\right\}. \end{aligned} \tag{15}$$

It can be shown that the solution to (15) is as follows: there exists a threshold value c^*, where

$$c^* \equiv \frac{\delta\pi}{(1-\delta)(2-\delta)}, \tag{16}$$

so that individual 1 vaccinates if $c_1 < c^*$ and does not vaccinate if $c_1 > c^*$; when $c_1 = c^*$, individual 1 is indifferent between the two choices.

6.2. Socially optimal vaccination

For the social planner's problem, we need to distinguish between the state $(2,0)$ when the two infected individuals are individuals 1 and 3 from the state $(2,0)$ when the two infected individuals are individuals 2 and 3. Let $W_{1,3}((2,0))$ denote the value of the state $(2,0)$ when the two infected individuals are individuals 1 and 3, and let $W_{2,3}((2,0))$ denote the value of the state $(2,0)$ when the two infected individuals are individuals 2 and 3. Using dynamic programming arguments, the planner's value function solves

$$W((1,0)) = \pi + \min\left\{\begin{array}{l} c_1 + \frac{\delta}{2}[W((2,1)) + W((1,1))] \\ \frac{\delta}{4}[W((1,0)) + W_{1,3}((2,0)) + W_{2,3}((2,0)) + W((3,0))] \end{array}\right., \tag{17}$$

$$W((3,0)) = \frac{3\pi}{1-\delta}, \tag{18}$$

$$W_{1,3}((2,0)) = 2\pi + \delta W((3,0)) = \frac{3\pi}{1-\delta} - \pi, \tag{19}$$

$$W_{2,3}((2,0)) = 2\pi + \min\{c_1 + \delta W(2,1), \delta W(3,0)\}, \tag{20}$$

$$W((1,1)) = \pi + \frac{\delta}{2}[W((1,1)) + W((2,1))], \tag{21}$$

$$W((2,1)) = 2\pi + \delta W((2,1)) = \frac{2\pi}{1-\delta}. \tag{22}$$

In the expression for $W((1,0))$, the first argument is the value of vaccinating individual 1, while the second argument is the value of not vaccinating individual 1. Note that $W((1,1))$ is the planner's value when individual 2 is the only susceptible individual. As before, suppose that $c_1 \le \delta\pi/(1-\delta)$. Using (18) and (22), the solution to (20) is to vaccinate individual 1 when individuals 2 and 3 are infected. This yields $W_{2,3}((2,0)) = 2\pi/(1-\delta)+c_1$. Note that (21) and (22) give us $W((1,1)) = 2\pi/(1-\delta)(2-\delta)$. It is straightforward—though somewhat tedious—to show that the solution to (17) is as follows: there exists a threshold value c^{**}, where

$$c^{**} \equiv \frac{\delta\pi(4-\delta-\delta^2)}{2(2-\delta)^2(1-\delta)},$$

such that the social planner chooses to vaccinate individual 1 if $c_1 < c^{**}$, does not vaccinate individual 1 if $c_1 > c^{**}$ and is indifferent between vaccinating and not vaccinating individual 1 when $c_1 = c^{**}$. The social planner's threshold c^{**} is higher than the threshold value c^* that individual 1 uses in equilibrium (see (16)). This means, in particular, that if $c_1 \in (c^*, c^{**})$, then what individual 1 chooses in equilibrium (namely not to vaccinate) is different from what the social planner would choose (namely to vaccinate).

7. The model with inflexible vaccination timing

We now relax Assumption A4 and consider the effects of inflexibility in vaccination timing. As noted in the Introduction, there are several analyses in the literature in which the planner and the individuals are only able to vaccinate once at the outset and are unable to revise their decision at later stages (in case vaccination was not taken). These include those by Brito et al. (1991), Geoffard and Philipson (1997), Xu (1999), Barrett (2003), Sadique et al. (2005), Heal and Kunreuther (2005), Kureishi (2009), Boulier (2009) and Galeotti and Rogers (2013). Inflexible vaccination timing is a modeling assumption that is typically imposed for simplicity, as it in some instances reduces complicated dynamic games to simpler static games.

As in previous sections, the key to understanding the effects of inflexible vaccination timing is that it induces strategic interaction, thereby driving a wedge between first-best and equilibrium outcomes.

Assume that individuals have only one opportunity to get vaccinated, namely at time 0.[18] As before, we assume that the state of the population at time 0 is $(1,0)$, i.e., there is one infected individual and two susceptible individuals.

Proposition 5. *The equilibrium pattern of vaccination may be socially suboptimal if vaccination timing is inflexible.*

7.1. Equilibrium vaccination

As before, $\underline{U}_{i,t} = \pi/(1-\delta)$ for all t. Since the opportunity to get vaccinated exists only at $t=0$, σ_i must satisfy the condition $\sigma_i(\omega;t)=0$ for all $t>0$ and $\omega \in \Omega_S^i$, $i=1,2$, and the individuals' decision (4) has to be modified to account for this restriction. First, consider any period $t>0$, in which vaccination is not an option. We must have, for $(I,P) \in \Omega_S^i$,

$$U_{i,t}((I,P);\sigma_j) = \delta\left[\frac{I}{2}\underline{U}_{i,t+1} + \left(1-\frac{I}{2}\right)\times \sum_{\omega' \in \Omega_S^i} Q_{NV}(\omega'|(I,P);\sigma_j)U_{i,t+1}(\omega';\sigma_j)\right].$$

Since individuals are infinitely-lived, $U_{i,t}(\omega;\sigma_j) = U_i(\omega;\sigma_j)$ for all $t>0$ and $\omega \in \Omega_S^i$. The transition probability function $Q_{NV}(\omega'|(I,P);\sigma_j)$ given the restriction $\sigma_j(\omega;t)=0$ for all $t>0$ is specified by the transition matrix

	$(2,0)$	$(1,0)$	$(1,1)$
$(1,0)$	$\frac{1}{2}$	$\frac{1}{2}$	0
$(1,1)$	0	0	1

It can be shown that

$$U_i((1,1);\sigma_j) = \frac{\delta\pi}{(1-\delta)(2-\delta)},$$

$$U_i((2,0);\sigma_j) = \frac{\delta\pi}{1-\delta},$$

$$U_i((1,0);\sigma_j) = \frac{\delta\pi(2+\delta)}{(1-\delta)(4-\delta)}.$$

At time 0, when the state is $(1,0)$, susceptible individual i's decision problem is

$$\begin{cases} \min\left\{c, 0+\delta\left(\frac{1}{2}\underline{U}_{i,1}+\frac{1}{2}U_i((1,1);\sigma_j)\right)\right\} & \text{if } \sigma_j((1,0),0)=1 \\ \min\left\{c, 0+\delta\left(\frac{1}{2}\underline{U}_{i,1}+\frac{1}{2}(\frac{1}{2}U_i((1,0);\sigma_j)\right.\right. & \\ \left.\left.+\frac{1}{2}U_i((2,0);\sigma_j))\right)\right\} & \text{if } \sigma_j((1,0),0)=0 \end{cases}.$$

The equilibrium is given as follows: if $c \le \delta\pi/(1-\delta)(2-\delta)$, then both susceptible individuals vaccinate at time 0; if $\delta\pi/(1-\delta)(2-\delta) \le c \le \delta\pi(2+\delta)/(1-\delta)(4-\delta)$, then only one susceptible individual vaccinates at time 0; last, if $c \ge \delta\pi(2+\delta)/(1-\delta)(4-\delta)$, then no individual vaccinates at time 0. In this setting, therefore, a susceptible individual's decision can depend on the action of the other susceptible individual. This is the case when $c \in [\delta\pi/(1-\delta)(2-\delta), \delta\pi(2+\delta)/(1-\delta)(4-\delta)]$.

7.2. Socially optimal vaccination

Let us modify the planner's problem (6) to incorporate the restriction that the vaccination opportunity exists only at $t=0$. For any $t>0$, when vaccination is not possible, the value of the planner's problem must solve

$$W_t((I,P)) = I\pi + \delta \sum_{\omega' \in \Omega} Q(\omega'|(I,P),0)W_{t+1}(\omega') \tag{23}$$

for all $(I,P) \in \Omega$. The transition function $Q(\cdot|\cdot,0)$ is given as follows:

	$(3,0)$	$(2,0)$	$(2,1)$	$(1,0)$	$(1,1)$	$(1,2)$
$(3,0)$	1	0	0	0	0	0
$(2,0)$	1	0	0	0	0	0
$(2,1)$	0	0	1	0	0	0
$(1,0)$	$\frac{1}{4}$	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0
$(1,1)$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0
$(1,2)$	0	0	0	0	0	1

Since individuals are infinitely-lived, we have $W_t(\omega) = W(\omega)$ for all $t>0$ and $\omega \in \Omega$. It can be verified, using (23) and the transition matrix above, that

$$W((1,0)) = \frac{(4+3\delta+2\delta^2)\pi}{(1-\delta)(4-\delta)},$$

$$W((2,0)) = \frac{(2+\delta)\pi}{1-\delta},$$

$$W((1,1)) = \frac{2\pi}{(1-\delta)(2-\delta)},$$

$$W((3,0)) = \frac{3\pi}{1-\delta},$$

$$W((2,1)) = \frac{2\pi}{1-\delta},$$

$$W((1,2)) = \frac{\pi}{1-\delta}.$$

[18] Alternatively, one could introduce inflexibility by assuming that vaccination is only available every nth period or that it is available in each period with some probability $p \in (0,1)$.

At $t=0$, when the state of the population is $(1,0)$ and the opportunity to vaccinate susceptible individuals exists, the planner's problem is

$$\min\begin{cases}\pi+\delta\Big(\frac{1}{2}\ \Big(\frac{1}{2}\ W((1,0))+\frac{1}{2}W((2,0))\Big)\\ \quad+\frac{1}{2}\ \Big(\frac{1}{2}\ W((2,0))+\frac{1}{2}W((3,0))\Big)\Big)\\ \pi+c+\delta\Big(\frac{1}{2}\ W((1,1))+\frac{1}{2}W((2,1))\Big)\\ \pi+2c+\delta W((1,2))\end{cases}. \tag{24}$$

The solution to the social planner's problem is given as follows: if $c\le\delta\pi/(1-\delta)(2-\delta)$, then it is optimal to vaccinate both susceptible individuals at time 0; if $\delta\pi/(1-\delta)(2-\delta)\le c\le\delta\pi(4+\delta-2\delta^2)/(1-\delta)(2-\delta)(4-\delta)$, then it is optimal to vaccinate one susceptible individual at time 0; last, if $c\ge\delta\pi(4+\delta-2\delta^2)/(1-\delta)(2-\delta)(4-\delta)$, then it is optimal to not vaccinate any individual at time 0. Since $\delta\pi(4+\delta-2\delta^2)/(1-\delta)(2-\delta)(4-\delta)>\delta\pi(2+\delta)/(1-\delta)(4-\delta)$, if $\delta\pi(2+\delta)/(1-\delta)(4-\delta)<c<\delta\pi(4+\delta-2\delta^2)/(1-\delta)(2-\delta)(4-\delta)$, then in equilibrium no one vaccinates even though it is socially optimal for one susceptible individual to get vaccinated.

8. The model with a finite horizon

We now relax Assumption A5 and consider the model under a finite planning horizon. While finite horizon models are routinely used for solving the planner's problem in the mathematical biology literature, to our knowledge ours is the first exploration of the effects that the planning horizon may have on the social optimality of equilibrium vaccine uptake. As noted in the Introduction, when the horizon is finite, the model features strategic interactions. As we will show, this can in turn create the scope for non-internalized external effects from vaccination.

Turning to the analysis, we assume that each individual lives for three periods ($t=0,1,2$). At the beginning of period 0, individual 3 is infected, while the other two individuals are susceptible. For simplicity, assume that the discount factor δ is 1.

Proposition 6. *The equilibrium pattern of vaccination may be socially suboptimal if the horizon is finite.*

8.1. Equilibrium vaccination

Since the model applies only to periods 0, 1, and 2, the following hold: for all $t>2$, $\underline{U}_{i,t}=0$, $U_{i,t}(\omega;\sigma_j)=0$ for all $\omega\in\Omega_S^i$, and $V_{i,t}(\omega;\sigma_j)=0$ for all $\omega\in\Omega_P^i$. Furthermore, the assumption that the vaccine confers perfect immunity implies that $V_{i,1}(\omega;\sigma_j)=V_{i,2}(\omega;\sigma_j)=0$ for all $\omega\in\Omega_P^i$. Given these restrictions, a susceptible individual i's decision problem is as follows:

$$U_{i,0}((1,0);\sigma_j)=\begin{cases}\min\Big\{c,\frac{1}{2}(2\pi)+\frac{1}{2}U_{i,1}((1,1);\sigma_j)\Big\} & \text{if } \sigma_j((1,0),0)=1\\ \min\Big\{c,\frac{1}{2}(2\pi)+\frac{1}{2}(\frac{1}{2}U_{i,1}((1,0);\sigma_j) & \\ \quad+\frac{1}{2}U_{i,1}((2,0);\sigma_j))\Big\} & \text{if } \sigma_j((1,0),0)=0\end{cases},$$

$$U_{i,1}((1,0);\sigma_j)=\min\Big\{c,\frac{1}{2}\pi\Big\},$$

$$U_{i,1}((1,1);\sigma_j)=\min\Big\{c,\frac{1}{2}\pi\Big\},$$

$$U_{i,1}((2,0);\sigma_j)=\min\{c,\pi\},$$

$$U_{i,2}(\omega;\sigma_j)=\min\{c,0\}=0\quad\text{for all }\omega\in\Omega_S^i.$$

Unlike in the infinite-horizon setting, there are now strategic interactions since whether a susceptible individual chooses to get vaccinated or not can depend on whether someone else decides to vaccinate or not. To see this, suppose $c>\pi$, which yields $U_{i,1}((1,0);\sigma_j)=U_{i,1}((1,1);\sigma_j)=\frac{1}{2}\pi$ and $U_{i,1}((2,0);\sigma_j)=\pi$. If, furthermore, $c\in(\frac{5}{4}\pi,\frac{11}{8}\pi)$, then we have $c>\frac{1}{2}(2\pi)+\frac{1}{2}U_{i,1}((1,1);\sigma_j)$ and $c<\frac{1}{2}(2\pi)+\frac{1}{2}(\frac{1}{2}U_{i,1}((1,0);\sigma_j)+\frac{1}{2}U_{i,1}((2,0);\sigma_j))$. This means that in period 0, it is optimal for individual i to vaccinate when $\sigma_j((1,0),0)=0$, and not to do so when $\sigma_j((1,0),0)=1$.

8.2. Socially optimal vaccination

Our finite-horizon restriction means that $W_t(\omega)=0$ for all $t>2$. The social planner's problem is given below.

$$W_0((1,0))=\pi+\min\begin{cases}\frac{1}{4}W_1((1,0))+\frac{1}{2}W_1((2,0))+\frac{1}{4}W_1((3,0))\\ c+\frac{1}{2}W_1((1,1))+\frac{1}{2}W_1((2,1))\\ 2c+W_1((1,2))\end{cases},$$

$$W_1((1,0))=\pi+\min\begin{cases}\frac{1}{4}W_2((1,0))+\frac{1}{2}W_2((2,0))+\frac{1}{4}W_2((3,0))\\ c+\frac{1}{2}W_2((1,1))+\frac{1}{2}W_2((2,1))\\ 2c+W_2((1,2))\end{cases},$$

$$W_1((1,1))=\pi+\min\Big\{\frac{1}{2}W_2((1,1))+\frac{1}{2}W_2((2,1)),c+W_2((1,2))\Big\},$$

$$W_1((1,2))=\pi+W_2((1,2)),$$
$$W_1((2,0))=2\pi+\min\{W_2((3,0)),c+W_2((2,1))\},$$
$$W_1((2,1))=2\pi+W_2((2,1)),$$
$$W_1((3,0))=3\pi+W_2((3,0)),$$
$$W_2((I,P))=I\pi\quad\text{for all }(I,P)\in\Omega.$$

The following example shows that the equilibrium need not coincide with the solution to the planner's problem.

8.3. Inefficiency of equilibrium: Example 3

Suppose that $\frac{3}{2}\pi>c>\frac{11}{8}\pi$. Let us first consider the equilibrium outcome. In this case, $U_{i,1}((1,0);\sigma_j)=U_{i,1}((1,1);\sigma_j)=\frac{1}{2}\pi$ and $U_{i,1}((2,0);\sigma_j)=\pi$, which imply that neither individual 1 nor individual 2 would choose to get vaccinated in state $(1,0)$ in period 0. Now, consider the solution to the planner's problem. In this case, we have that

$$W_1((1,1))=\min\Big\{\frac{5}{2}\pi,2\pi+c\Big\}=\frac{5}{2}\pi,$$

$$W_1((1,0))=\min\Big\{3\pi,\frac{5}{2}\pi+c,2\pi+2c\Big\}=3\pi,$$

$$W_1((2,0))=\min\{5\pi,4\pi+c\}=5\pi,$$
$$W_1((3,0))=6\pi,$$
$$W_1((2,1))=4\pi,$$
$$W_1((1,2))=2\pi,$$

$$W_0((1,0))=\min\Big\{\frac{23}{4}\pi,\frac{17}{4}\pi+c,3\pi+2c\Big\}=\frac{17}{4}\pi+c.$$

Therefore, in state $(1,0)$ in period 0, the planner would choose to vaccinate one susceptible individual.

9. Conclusion

In this paper, we have considered the social optimality of equilibrium vaccination patterns under decentralized decision making. We find that the welfare theorem of Francis (1997) may not be robust to changes in the sufficient conditions set out in his work. Specifically, we relax the assumptions of homogeneity, infinite planning horizon, no recovery from infection, perfect vaccines and perfectly flexible vaccination timing. We find that when relaxing any of these assumptions, equilibrium vaccine uptake may be different than that chosen by a benevolent social planner. This stems from the fact that the Francis welfare theorem is a consequence of (a) ex ante homogeneity and (b) the absence of strategic interaction in vaccination decisions.

As we have shown in several different ways in this paper, in general, one should expect that some welfare loss will be the outcome of fully decentralized vaccination decisions. Having said that, in order to make any sensible statements about policy interventions based on this insight, one should carefully consider the disease at hand and determine which of the departures from the Francis framework applies to the particular disease or policy environment. Simply relying on the notion that “individuals under-vaccinate and therefore there is scope for policy intervention” is not sufficient, as the Francis analysis shows. It should also be food for thought that the Francis framework is in a sense the “natural” dynamic model to show that the market leads to under-vaccination, although we have seen that it is in fact inadequate for this purpose. At the very least, it shows that one should be cautious in relying on intuitions about the welfare properties of vaccination markets without resorting to explicit analysis.

Turning to possible corrective policies, the inefficiencies in decentralized vaccine uptake that we have identified can be overcome by offering individuals suitable state-dependent Pigouvian subsidies.[19] But while such subsidies can in principle perfectly align private and public incentives to vaccinate against a disease, they may be difficult to implement in practice. This is because the subsidy on offer at any given point in time must depend on aggregate disease prevalence at that time. In heterogeneous populations, the optimal Pigouvian subsidy to an individual may furthermore depend on the characteristics of the individual in question and on the identity of the infected individuals. In practice, simpler schemes must be considered, which limits how far subsidies can go in decentralizing the socially optimal vaccination policy.

Although for ease of exposition our analysis considered models with only three individuals, our main results regarding the inefficiencies of equilibrium vaccination patterns, as well as the reasons behind these inefficiencies, extend to settings with any number of individuals. Ultimately, the inefficiency of equilibrium vaccination arises from external effects and the strategic interdependence of individuals' vaccination decisions, and the existence of these are wholly independent of assumptions concerning how many individuals there are in the vaccination market. Note also that, for any $N>3$, it is easy to extend our modeling framework to a market with N individuals. The only changes that need to be made are (i) changing the 2 in the denominator of equations (2), (3), and (5) to $N-1$; and (ii) defining the probability transition functions Q_V, Q_{NV}, and Q appropriately.

Last, we should like to make a few comments about how our analysis contributes to the debate on whether public intervention in vaccine markets is warranted on efficiency grounds. It is clear that the case against any public intervention, at least to the extent that it relies on the Francis neutrality result, is weakened as a result of our findings. While it is difficult a priori to point out which of the sufficient assumptions for the welfare theorem is violated in a given situation (i.e., for a given population, disease and vaccine combination), it is hard to accept the supposition that none of them are violated. On the face of it, this suggests that there is indeed a case for interventions that align private and public incentives for vaccination uptake. Having said that, we should also emphasize that while the Francis assumptions are sufficient for the neutrality result to hold, they are in fact not necessary. There are many instances in which the sufficient assumptions are violated but where the neutrality result still holds. It is ultimately an empirical question, whether the environment and the parameters of the problem are such that decentralized vaccination decisions are socially efficient. In short, the extent to which public intervention in vaccine markets is warranted on efficiency grounds is not something that can be determined purely through formal analysis. As such, any case for or against such intervention must draw not only on analysis but also on a detailed knowledge of the environment.

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Appendix A. Spontaneous recovery

A.1. Socially optimal vaccination

The transition probability function Q for the social planner's problem is given by the following transition matrix (because of the size of this matrix, it is broken up into two parts):

	$(3,0)$	$(2,0)$	$(2,1)$	$(1,0)$
$(3,0)$	$(1-\rho)^3$	0	$3\rho(1-\rho)^2$	0
$(2,0)$ if $n=0$	$(1-\rho)^2$	0	$2\rho(1-\rho)$	0
$(2,0)$ if $n=1$	0	0	$(1-\rho)^2$	0
$(2,1)$	0	0	$(1-\rho)^2$	0
$(1,0)$ if $n=0$	$\frac{1}{4}(1-\rho)$	$\frac{1}{2}(1-\rho)$	$\frac{1}{4}\rho$	$\frac{1}{4}(1-\rho)$
$(1,0)$ if $n=1$	0	0	$\frac{1}{2}(1-\rho)$	0
$(1,0)$ if $n=2$	0	0	0	0
$(1,1)$ if $n=0$	0	0	$\frac{1}{2}(1-\rho)$	0
$(1,1)$ if $n=1$	0	0	0	0
$(1,2)$	0	0	0	0
$(0,3)$	0	0	0	0
$(0,2)$	0	0	0	0
$(0,1)$	0	0	0	0

	$(1,1)$	$(1,2)$	$(0,3)$	$(0,2)$	$(0,1)$
$(3,0)$	0	$3\rho^2(1-\rho)$	ρ^3	0	0
$(2,0)$ if $n=0$	0	ρ^2	0	0	0
$(2,0)$ if $n=1$	0	$2\rho(1-\rho)$	ρ^2	0	0
$(2,1)$	0	$2\rho(1-\rho)$	ρ^2	0	0
$(1,0)$ if $n=0$	$\frac{1}{2}\rho$	0	0	0	$\frac{1}{4}\rho$
$(1,0)$ if $n=1$	$\frac{1}{2}(1-\rho)$	$\frac{1}{2}\rho$	0	$\frac{1}{2}\rho$	0
$(1,0)$ if $n=2$	0	$1-\rho$	ρ	0	0
$(1,1)$ if $n=0$	$\frac{1}{2}(1-\rho)$	$\frac{1}{2}\rho$	0	$\frac{1}{2}\rho$	0
$(1,1)$ if $n=1$	0	$1-\rho$	ρ	0	0
$(1,2)$	0	$1-\rho$	ρ	0	0
$(0,3)$	0	0	1	0	0
$(0,2)$	0	0	0	1	0
$(0,1)$	0	0	0	0	1

A.2. Example 1

Given $\delta\pi/(1-\delta(1-\rho))>c>\delta\pi/(1-\delta(1-\rho))(2-\delta(1-\rho))$, we have $U_i((1,1);\sigma_j)=\delta\pi/(1-\delta(1-\rho))(2-\delta(1-\rho))$, $U_i((2,0);\sigma_j)=c$, and, given $\sigma_j(1,0)=0$,

[19] See Rowthorn and Toxvaerd (2012) for such a subsidy scheme in the context of treatment and (non-vaccine) prevention in the SIS (susceptible-infected-susceptible) model.

$$U_i((1,0);\sigma_j)=\min\left\{c,\frac{\delta(2c+4\pi-3c\delta-2\pi\delta-2c\rho+c\delta^2+3c\delta^2\rho^2-c\delta^2\rho^3+6c\delta\rho+3\pi\delta\rho-3c\delta\rho^2-3c\delta^2\rho)}{(4-\delta(1-\rho))(1-\delta(1-\rho))(2-\delta(1-\rho))}\right\}.$$

In state $(1,0)$, the value of not vaccinating is lower than the value of vaccinating when $c\geq\underline{c}$. Therefore, in equilibrium, no individual vaccinates in state $(1,0)$ when $c\geq\underline{c}$.

For the social planner's problem, $W((1,1))=\pi(2+\delta\rho)/(1-\delta(1-\rho))(2-\delta(1-\rho))$ and $W((2,0))=c+2\pi/(1-\delta(1-\rho))$. In state $(1,0)$, the value of vaccinating both susceptible individuals is $2c+\pi/(1-\delta(1-\rho))$, and the value of vaccinating only one susceptible individual is $c+\pi(2+\delta\rho)/(1-\delta(1-\rho))(2-\delta(1-\rho))$. With the given parameter restrictions, it is better to vaccinate only one susceptible individual than both susceptible individuals. This implies that, in state $(1,0)$, the social planner's problem reduces to choosing between vaccinating only one susceptible individual and vaccinating no one. The value of vaccinating no one is

$$\frac{1}{(1-\delta(1-\rho))(2-\delta(1-\rho))(4-\delta(1-\rho))}$$
$$\times[8\pi+4c\delta+2\pi\delta-6c\delta^2+2c\delta^3-3\pi\delta^2-6c\delta^2\rho^2$$
$$+6c\delta^3\rho^2-2c\delta^3\rho^3+\pi\delta^2\rho^2-4c\delta\rho+6\pi\delta\rho$$
$$+12c\delta^2\rho-6c\delta^3\rho+4\pi\delta^2\rho].$$

If $c\geq\overline{c}$, then it is better to vaccinate no one; if $c\leq\overline{c}$, then it is better to vaccinate only one susceptible individual.

Appendix B. Imperfect vaccine

B.1. Socially optimal vaccination

The transition probability function Q for the social planner's problem is given by the following transition matrix (because of the size of this matrix, it is broken up into two parts):

		$(3,0)$	$(2,0)$	$(2,1)$
$(3,0)$		1	0	0
$(2,0)$	if $n=0$	1	0	0
	if $n=1$	$1-\varepsilon$	0	ε
$(2,1)$		$1-\varepsilon$	0	ε
$(1,0)$	if $n=0$	$\frac{1}{4}$	$\frac{1}{2}$	0
	if $n=1$	$\frac{1}{4}(1-\varepsilon)$	$\frac{1}{4}(1-\varepsilon)$	$\frac{1}{2}\left(1-\frac{1}{2}(1-\varepsilon)\right)$
	if $n=2$	$\frac{1}{4}(1-\varepsilon)^2$	0	$(1-\varepsilon)\left(1-\frac{1}{2}(1-\varepsilon)\right)$
$(1,1)$	if $n=0$	$\frac{1}{4}(1-\varepsilon)$	$\frac{1}{4}(1-\varepsilon)$	$\frac{1}{2}\left(1-\frac{1}{2}(1-\varepsilon)\right)$
	if $n=1$	$\frac{1}{4}(1-\varepsilon)^2$	0	$(1-\varepsilon)\left(1-\frac{1}{2}(1-\varepsilon)\right)$
$(1,2)$		$\frac{1}{4}(1-\varepsilon)^2$	0	$(1-\varepsilon)\left(1-\frac{1}{2}(1-\varepsilon)\right)$

		$(1,0)$	$(1,1)$	$(1,2)$
$(3,0)$		0	0	0
$(2,0)$	if $n=0$	0	0	0
	if $n=1$	0	0	0
$(2,1)$		0	0	0
$(1,0)$	if $n=0$	$\frac{1}{4}$	0	0
	if $n=1$	0	$\frac{1}{2}\left(1-\frac{1}{2}(1-\varepsilon)\right)$	0
	if $n=2$	0	0	$\left(1-\frac{1}{2}(1-\varepsilon)\right)^2$
$(1,1)$	if $n=0$	0	$\frac{1}{2}\left(1-\frac{1}{2}(1-\varepsilon)\right)$	0
	if $n=1$		0	$\left(1-\frac{1}{2}(1-\varepsilon)\right)^2$
$(1,2)$		0	0	$\left(1-\frac{1}{2}(1-\varepsilon)\right)^2$

.

B.2. Example 2

With the given parameter values, if no individual ever chooses to get vaccinated, then

$$U_i((2,0);\sigma_j)=9, U_i((1,0);\sigma_j)=\frac{261}{31}, U_i((1,1);\sigma_j)=\frac{207}{25},$$
$$V_i((2,1);\sigma_j)=\frac{15}{2}, V_i((1,1);\sigma_j)=\frac{69}{10}, V_i((1,2);\sigma_j)=\frac{23}{2}.$$

To see that this is indeed an equilibrium, consider individual i's decision problem when individual i is susceptible.

- State $(2,0)\in\Omega_S^i$: The expected, discounted cost of vaccinating, $c+\delta[\varepsilon V_i((2,1);\sigma_j)+(1-\varepsilon)\underline{U}_i]$ (weakly) exceeds the expected, discounted cost of not vaccinating, $\delta\underline{U}_i$, when $c\geq\frac{3}{2}$.
- State $(1,1)\in\Omega_S^i$: The expected, discounted cost of vaccinating,
$$c+\delta\left[\frac{1}{2}(1-\varepsilon)\underline{U}_i+\left(1-\frac{1}{2}(1-\varepsilon)\right)\left[\frac{1}{2}(1-\varepsilon)V_i((2,1);\sigma_j)+\left(1-\frac{1}{2}(1-\varepsilon)\right)V_i((1,2);\sigma_j)\right]\right],$$
exceeds the expected, discounted cost of not vaccinating,
$$\delta\left[\frac{1}{2}\underline{U}_i+\frac{1}{2}\left(\frac{1}{2}(1-\varepsilon)U_i((2,0);\sigma_j)+\left(1-\frac{1}{2}(1-\varepsilon)\right)U_i((1,1);\sigma_j)\right)\right],$$
for all $c>0$.
- State $(1,0)\in\Omega_S^i$: Given that $\sigma_j((1,0))=0$, the expected, discounted cost of vaccinating for individual i,
$$c+\delta\left[\frac{1}{2}(1-\varepsilon)\underline{U}_i+\left(1-\frac{1}{2}(1-\varepsilon)\right)\left(\frac{1}{2}V_i((2,1);\sigma_j)+\frac{1}{2}V_i((1,1);\sigma_j)\right)\right],$$
exceeds the expected, discounted cost of not vaccinating,
$$\delta\left[\frac{1}{2}\underline{U}_i+\frac{1}{2}\left(\frac{1}{2}U_i((2,0);\sigma_j)+\frac{1}{2}U_i((1,0);\sigma_j)\right)\right],$$
when $c\geq\frac{471}{310}$.

Taken together, these results imply that if $c>\frac{471}{310}$, the no susceptible individual ever chooses to vaccinate in equilibrium.

For the planner's problem, if the only susceptible individual is vaccinated in state $(1,1)$, and no individual is vaccinated in any other state, then the value function W must be as follows:

$$W((1,0))=\frac{832}{31}, W((2,1))=\frac{55}{2}, W((3,0))=30,$$
$$W((1,2))=23, W((2,0))=29, W((1,1))=23+c.$$

Now, let us check that this is indeed the solution to the planner's problem.

- State $(2,0)$: The expected, discounted sum of all costs if the only susceptible individual is vaccinated, $c+\delta[(1-\varepsilon)W((3,0))+\varepsilon W((2,1))]$, exceeds the expected, discounted sum of all costs if no one is vaccinated, $\delta W((3,0))$, when $c\geq\frac{3}{2}$.
- State $(1,1)$: The expected, discounted sum of all costs if the only susceptible individual is vaccinated,
$$c+\frac{\delta}{4}(1-\varepsilon)^2W((3,0))+\frac{\delta}{2}(1-\varepsilon)\left(1-\frac{1}{2}(1-\varepsilon)\right)W((2,1))$$
$$+\delta\left(1-\frac{1}{2}(1-\varepsilon)\right)\frac{1}{2}(1-\varepsilon)W((2,1))+\delta\left(1-\frac{1}{2}(1-\varepsilon)\right)^2W((1,2)),$$

is (weakly) lower than the expected, discounted sum of all costs if no one is vaccinated,

$$\frac{\delta}{4}(1-\varepsilon)W((3,0))+\frac{\delta}{2}\left(1-\frac{1}{2}(1-\varepsilon)\right)W((2,1))$$
$$+\frac{\delta}{4}(1-\varepsilon)W((2,0))+\delta\left(1-\frac{1}{2}(1-\varepsilon)\right)\frac{1}{2}W((1,1)),$$

when $c \le \frac{109}{50}$.

- State $(1,0)$: The expected, discounted sum of all costs of vaccinating one susceptible individual,

$$c+\frac{\delta}{4}(1-\varepsilon)W((3,0))+\frac{\delta}{4}(1-\varepsilon)W((2,0))$$
$$+\delta\left(1-\frac{1}{2}(1-\varepsilon)\right)\frac{1}{2}W((2,1))+\delta\left(1-\frac{1}{2}(1-\varepsilon)\right)\frac{1}{2}W((1,1)),$$

exceeds the cost of not vaccinating anyone, $\delta[\frac{1}{4}W((1,0))+\frac{1}{4}W((3,0))+\frac{1}{2}W((2,0))]$, when $c \ge \frac{6141}{3410}$. Also, the expected, discounted sum of all costs of vaccinating two susceptible individuals,

$$2c+\frac{\delta}{4}(1-\varepsilon)^2W((3,0))+\delta\left(1-\frac{1}{2}(1-\varepsilon)\right)^2W((1,2))$$
$$+2\delta\left(\frac{1}{2}(1-\varepsilon)\left(1-\frac{1}{2}(1-\varepsilon)\right)\right)W((2,1)),$$

exceeds the expected, discounted sum of all costs of not vaccinating anyone when $c \ge \frac{119}{62}$.

Taken together, these results imply that, for $c \in (\frac{119}{62}, \frac{109}{50})$, the solution to the planner's problem is to vaccinate the only susceptible individual in state $(1,1)$, and not to vaccinate anyone in all other states.

References

Althouse, B.M., Bergstrom, T.C., Bergstrom, C.T., 2010. A public choice framework for controlling transmissible and evolving diseases. Proc. Natl. Acad. Sci. 107 (1), 1696–1701.

Auld, M.C., 2003. Choice, beliefs, and infectious disease dynamics. J. Health Econ. 22 (3), 361–377.

Barrett, S., 2003. Global disease eradication. J. Eur. Econ. Assoc. 1 (2–3), 591–600.

Bauch, C.T., 2005. Imitation dynamics predict vaccinating behaviour. Proc. R. Soc. B 272, 1669–1675.

Bauch, C.T., Earn, D.J.D., 2004. Vaccination and the theory of games. Proc. Natl. Acad. Sci. 101 (36), 13391–13394.

Bauch, C., Onofrio, A.d., Manfredi, P., 2013. Behavioral epidemiology of infectious diseases: an overview. In: d.Onofrio, A., Manfredi, P. (Eds.), Modeling the Interplay between Human Behavior and Spread of Infectious Diseases. Springer, New York.

Behncke, H., 2000. Optimal control of deterministic epidemics. Optim. Control Appl. Methods 21 (6), 269–285.

Boulier, B.L., 2009. A Shot in the Dark: Uncertainty and Vaccine Demand and Supply. Mimeo.

Boulier, B.L., Datta, T.S., Goldfarb, R.S., 2007. Vaccination externalities. B.E. J. Econ. Anal. Policy 7 (1) (Contributions), Article 23.

Brito, D.L., Sheshinski, E., Intriligator, M.D., 1991. Externalities and compulsory vaccinations. J. Public Econ. 45 (1), 69–90.

Chen, F., 2006. A susceptible-infected epidemic model with voluntary vaccinations. J. Math. Biol. 53 (2), 253–272.

Chen, F., Cottrell, A., 2009. Dynamic equilibria in an epidemic model with voluntary vaccinations. J. Biol. Dyn. 3 (4), 357–375.

Cojocaru, M.G., Bauch, C.T., Johnston, M.D., 2007. Dynamics of vaccination strategies via projected dynamical systems. Bull. Math. Biol. 69 (5), 1453–1476.

Fenichel, E.P., 2013. Economic considerations for social distancing and behavioral based policies during an epidemic. J. Health Econ. 32 (2), 440–451.

Fenichel, E.P., Castillo-Chavez, C., Ceddia, M.G., Chowell, G., Gonzalez Parra, P.A., Hickling, G.J., Holloway, G., Horan, R., Morin, B., Perrings, C., Springborn, M., Velazquez, L., Villalobos, C., 2011. Adaptive human behavior in epidemiological models. Proc. Natl. Acad. Sci. 108 (15), 6306–6311.

Fenichel, E.P., Wang, X., 2013. The mechanism and phenomenon of adaptive human behavior during an epidemic and the role of information. In: d'Onofrio, A., Manfredi, P. (Eds.), Modeling the Interplay between Human Behavior and Spread of Infectious Diseases. Springer, New York.

Fine, P.E.M., Clarkson, J.A., 1986. Individual versus public priorities in the determination of optimal vaccination policies. Am. J. Epidemiol. 124 (6), 1012–1020.

Forslid, R., 2006. Can We Trust Monopolistic Firms as Suppliers of Vaccines for the Avian Influenza? Mimeo.

Forslid, R., Herzing, M. 2008. On the Optimal Production Capacity for Influenza Vaccine. Mimeo.

Francis, P.J., 2004. Optimal tax/subsidy combinations for the flu season. J. Econ. Dyn. Control 28 (10), 2037–2054.

Francis, P.J., 1997. Dynamic epidemiology and the market for vaccinations. J. Public Econ. 63 (3), 383–406.

Francis, P.J., 2007. Dynamic Vaccination Externalities with Heterogeneity, Recovery and Discounting. Mimeo.

Galeotti, A., Rogers, B., 2013. Immunizations and group structure. Am. Econ. J.: Microecon. 5 (2), 1–32.

Geoffard, P.-Y., Philipson, T., 1997. Disease eradication: private versus public vaccination. Am. Econ. Rev. 87 (1), 222–230.

Gersovitz, M., 2003. Births, recoveries, vaccinations and externalities. In: Stiglitz, E., Arnott, R.J. (Eds.), Economics for an Imperfect World: Essays in Honor of JosephHonor of Joseph E. Stiglitz. MIT Press, Cambridge, pp. 469–483.

Gersovitz, M., 2011. The economics of infection control. Annu. Rev. Resour. Econ. 3, 277–296.

Gersovitz, M., Hammer, J.S., 2003. Infectious diseases, public policy, and the marriage of economics and epidemiology. World Bank Res. Obs. 18 (2), 129–157.

Goyal, S. and A. Vigier (2014): Interaction, Protection and Epidemics, Cambridge-INET Institute Working Paper Series No: 2014/09.

Hansen, E., Day, T.J., 2011. Optimal control of epidemics with limited resources. J. Math. Biol. 62 (3), 423–451.

Heal, G., Kunreuther, H., 2005. The Vaccination Game. Mimeo.

Hethcote, H.W., Waltman, P., 1973. Optimal vaccination schedules in a deterministic epidemic model. Math. Biosci. 18 (3–4), 365–381.

Keogh-Brown, M.R., Wren-Lewis, S., Edmunds, W.J., Beutels, P., Smith, R.D., 2010. The possible macroeconomic impact on the UK of an influenza pandemic. Health Econ. 19 (11), 1345–1360.

Kessing, S., Nuscheler, R., 2003. Monopoly pricing with negative network effects: the case of vaccines. Discussion Paper SP II 2003-06, Wissenschaftszentrum Berlin.

Klein, E., Laxminarayan, R., Smith, D.L., Gilligan, C.A., 2007. Economic incentives and mathematical models of disease. Environ. Dev. Econ. 12 (5), 707–732.

Kremer, M., 2000a. Creating markets for new vaccines, Part I: rationale, in innovation policy and the economy, vol. 1. National Bureau of Economic Research, pp. 35–72.

Kremer, M., 2000b. Creating markets for new vaccines, Part II: design issues, in innovation policy and the economy, vol. 1. National Bureau of Economic Research, pp. 73–118.

Kremer, M., Snyder, C.M., 2003. Why are drugs more profitable than vaccines? NBER Working Paper # 9833.

Kremer, M., Snyder, C.M., Williams, H., 2008. Which Diseases Generate the Largest Epidemiological Externalities? Mimeo.

Kureishi, W., 2009. Partial vaccination programs and the eradication of infectious diseases. Econ. Bull. 29 (4), 2758–2769.

Liu, J., Kochin, B.F., Tekle, Y.I., Galvani, A.P., 2012. Epidemiological game-theory dynamics of chickenpox vaccination in the USA and Israel. J. R. Soc. Interface 9 (66), 68–76.

Maskin, E., Tirole, J., 2001. Markov perfect equilibrium, I: observable actions. J. Econ. Theory 100 (2), 191–219.

Mechoulan, S., 2005. Market Structure and Communicable Diseases. University of Toronto, Department of Economics Working Paper # tecipa-241.

Medlock, J., Luz, P.M., Struchiner, C.J., Galvani, A.P., 2009. The impact of transgenic mosquitoes on dengue virulence to humans and mosquitoes. Am. Nat. 174 (4), 565–577.

Morton, R., Wickwire, K.H., 1974. On the optimal control of a deterministic epidemic. Adv. Appl. Probab. 6 (4), 622–635.

d'Onofrio, A., Manfredi, P., Salinelli, E., 2013. Vaccinating behavior and the dynamics of vaccine preventable infections. In: d'Onofrio, A., Manfredi, P. (Eds.), Modeling the Interplay between Human Behavior and Spread of Infectious Diseases. Springer, New York.

Philipson, T., 2000. Economic Epidemiology and Infectious Diseases. Handbook of Health Economics, vol. 1, Part B, pp. 1761–1799.

Reluga, T.C., 2010. Game theory of social distancing in response to an epidemic. PLoS Comput. Biol. 6 (5), e1000793.

Reluga, T.C., Galvani, A.P., 2011. A general approach for population games with application to vaccination. Math. Biosci. 230 (2), 67–78.

Rowthorn, R., Toxvaerd, F., 2012. The Optimal Control of Infectious Diseases via Prevention and Treatment. Mimeo.

Sadique, Z., Edmunds, W.J., Devlin, N., Parkin, D., 2005. Understanding Individuals' Decisions About Vaccination: A Comparison Between Expected Utility and Regret Theory Models. City University Economics Discussion Papers # 05/03.

Smith, R.D., Keogh-Brown, M.R., Barnett, T., Tait, J., 2009. The Economy-Wide Impact of Pandemic Influenza and the UK: A Computable General Equilibrium Modelling Experiment, BMJ 339:b4571.

Stiglitz, J.E., 1988. Economics of the Public Sector, Norton.

Stokey, N.L., Lucas Jr., R.E., Prescott, E.C., 1989. Recursive Methods in Economic Dynamics. Harvard University Press.

Toxvaerd, F., 2009. Recurrent Infection and Externalities in Prevention. Mimeo.

Veliov, V.M., 2005. On the effect of population heterogeneity on dynamics of epidemic diseases. J. Math. Biol. 51 (2), 123–143.

Veliov, V.M., 2008. Optimal control of heterogeneous systems: basic theory. J. Math. Anal. Appl. 346 (1), 227–242.

Wickwire, K.H., 1975. A note on the optimal control of carrier-borne epidemics. J. Appl. Probab. 12 (3), 565–568.

Xu, X., 1999. Technological improvements in vaccine efficacy and individual incentive to vaccinate. Econ. Lett. 65 (3), 359–364.