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Dynamic limit pricing

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I study a multiperiod model of limit pricing under one-sided incomplete information. I characterize pooling and separating equilibria and their existence and determine when these involve limit pricing. For some parameter constellations, the unique equilibrium surviving a D1 type refinement involves immediate separation on monopoly prices. For others, there are limit price equilibria surviving the refinement in which different types may initially pool and then (possibly) separate. Separation involves setting prices such that the inefficient incumbent's profits from mimicking are negative. As the horizon increases or as firms become more patient, limit pricing becomes increasingly difficult to sustain in equilibrium.

1. Introduction

■ Since Bain's (1949) pioneering work, limit pricing has been a staple of industrial economics. In a nutshell, limit pricing is the practice by which an incumbent firm (or cartel) deters potential entry to an industry by pricing below the profit maximizing price level. Early work on the subject took its cue from the casual observation that in some industries, firms price below the myopic profit maximizing price level *on a persistent basis*. This observation lead to the notion that by doing so, incumbent firms could somehow discourage potential entry which would otherwise have occurred, in effect by sacrificing profits in the short run in return for a maintenance of the monopoly position in the long run.

Bain (1949) noted that "[. . .] established sellers persistently or 'in the long run' forego prices high enough to maximize the industry profit for fear of thereby attracting new entry to the industry and thus reducing the demands for their outputs and their own profits."

The present work revisits received wisdom on equilibrium limit pricing in dynamic contexts, by way of a dynamic extension of a simple static model of one-sided incomplete information

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in the spirit of Milgrom and Roberts (1982). I demonstrate that although some aspects of the standard (static) analysis may be preserved qualitatively when moving to dynamic contexts, the quantitative results may radically differ. The analysis shows that when the horizon is sufficiently long and the players sufficiently patient, limit pricing becomes infeasible altogether.

In this article, I analyze a model of limit pricing with one-sided incomplete information in which a simple entry game is repeated as long as entry has not occurred. In this model, I identify two distinct regimes, a *monopoly price regime* and a *limit price regime*. In the monopoly price regime, limit price equilibria may exist but all such equilibria are ruled out by using a combination of equilibrium dominance and Cho and Kreps' (1987) criterion D1 at all information sets off the equilibrium path, as compared to a natural benchmark equilibrium in which the uninformed player makes use of all available information (in a sense that will be made precise). The unique equilibrium, using this notion, is one of immediate separation on monopoly prices. In the limit price regime, both pooling and separating equilibria may exist and all these involve limit pricing. I find that in the limit price regime, the basic logic of separating equilibria of a static single-round setting carries over to the separating equilibria of the dynamic setting. In particular, I find that by sacrificing enough at some (single) stage of the game, the efficient incumbent may credibly convey his identity to the entrant. Whether this signalling effectively precludes entry, and is thus worthwhile from the perspective of the incumbent firm, in turn depends on the entrant's incentives to enter. In the dynamic setting, as the future becomes more important, the relevant conditions needed to deter entry are increasingly unlikely to be satisfied. Specifically, I show that as the horizon becomes longer, it becomes more difficult to deter entry simply because the entrant's one-off cost of entry may not outweigh a long sequence of postentry profits, even if discounted. Similarly, I show that for a sufficiently patient entrant firm, an infinite sequence of discounted future profits will outweigh any bounded entry fee and thus, make entry inevitable. In both cases, adding dynamics to a static limit pricing model makes entry deterrence through limit pricing more difficult (or impossible) to sustain as an equilibrium outcome. Thus, immediate entry is likely to result, with each firm setting its respective monopoly price (regardless of the prevailing regime).

Although these results cast serious doubt on the viability of limit pricing, it should be mentioned that the basic trade-off found in the static analysis can be recovered in the dynamic setting, if one disregards the caveats above and assumes all incentive constraints to be satisfied. Even in this case, the dynamics of the model make somewhat unrealistic predictions. Specifically, one important difference with a static setting is that in the static setting, the benefits from deterring entry are bounded, whereas this is no longer the case in the dynamic setting, if the players are sufficiently patient. For a large enough discount factor and a sufficiently long horizon, the efficient incumbent needs to press the inefficient incumbent to make strictly negative profits from mimicking (e.g., by pricing below marginal cost). When the players are very patient, the short-term losses necessary to credibly signal to be a low-cost incumbent are unbounded.

Assuming that all the relevant feasibility constraints are satisfied, in the limit price regime, all equilibria satisfying the D1 type refinement (*anchored* D1) belong to a single class, consisting of (i) a (possibly nonzero and possibly infinite) number of periods during which the two types of incumbent pool; (ii) a period in which the efficient type engages in costly signalling, whereas the inefficient type reveals himself and invites entry; and (iii) continuation play in which the efficient type charges monopoly prices in all subsequent periods and deters entry, whereas the inefficient type competes against the entrant.

The welfare properties of these equilibria are not straightforward. It is true, as is the case in the static benchmark model, that in the period where separation takes place, welfare is unambiguously higher than it would be under symmetric information. This is because entry occurs under the same states of nature as under symmetric information, but the efficient type sets lower prices than would a monopolist. However, if separation is preceded by periods with pooling, the conclusion is less clear-cut. This is because pooling deters entry, which counterweights the benefits of lower prices set by the incumbent.[1] In the special case where the static pooling equilibrium yields lower welfare than under complete information, social welfare is unambiguously higher the earlier separation occurs. This is because immediate separation (which has good welfare properties) precludes periods with pooling (which have bad welfare properties). However, if instead, welfare under the static pooling equilibrium is higher than under complete information, then the welfare comparison of equilibria that differ in their timing of separation becomes impossible, without making explicit assumptions about parameters and the nature of postentry competition. This is because one must then compare magnitudes of positive payoffs rather than comparing signs of payoffs.[2]

□ **Empirical evidence of limit pricing.** Direct evidence of limit pricing is difficult to come by, for several reasons. First, limit pricing is a response to the *threat* of entry rather than to *actual* entry, as emphasized by Goolsbee and Syverson (2008). The threat of entry is typically not observed but must instead be inferred from context, as the few existing empirical articles on limit pricing do. Second, if successfully carried out, limit pricing causes potential entrants to stay out of the industry. Again, it is not straightforward to identify the "absence of entry," whereas in instances of predatory pricing, an active firm is easily identified as leaving the industry. Third, even pricing patterns that are broadly consistent with the basic idea of limit pricing may be the result of other types of dynamic demand linkages, such as the building up of a loyal customer base (see, e.g., Bain, 1949; Gedge, Roberts, and Sweeting, 2013).[3]

More recently, the literature has identified two industries in which the threat of entry can be inferred from context, namely, the airline industry and the cable TV industry. Entry into the airline industry has been analyzed by Goolsbee and Syverson (2008), Morrison (2001), and Gedge, Roberts, and Sweeting (2013). In this literature, the threat of entry on a route between two airports A and B is identified with the presence of an entrant airline in airports A and B separately. That is, if an entrant airline is already serving the route A-C and now starts operating on route B-D, then the presence of the entrant on both endpoints of the route A-B is taken as a threat that it will start operating on this route as well. The results of Goolsbee and Syverson (2008), although suggestive (or at least consistent with) limit pricing, are inconclusive. Gedge, Roberts, and Sweeting (2013) also study the airline industry to conclude that strategic considerations are in play.

In the cable TV industry, Seamans (2013) uses a similar approach, by exploiting the fact that entering (and offering cable TV services to) a geographical area, is significantly more appealing for a company that already serves an adjacent area (because fixed costs can be shared). Thus, the threat of entry of an entrant can be identified as the physical proximity of the entrant's existing foothold to the market in question. The analysis of Seamans (2013) concludes that the evidence from the cable TV industry is consistent with limit pricing.

Although the analysis of these industries seems to lend some credence to the practice of limit pricing, one has to be cautious in interpreting this evidence. The reason is that in both cases, both entrant and incumbent firms are engaged in multimarket contact. In other words, each entry scenario is but a small part of a larger game played by the firms across different geographical locations and markets. This type of competitive environment does not fit neatly into the standard entry deterrence framework, as exemplified by the Milgrom and Roberts (1982) model. For example, if an incumbent signals strength via limit pricing on a given route, what does that imply for the entrant's entry decisions on other routes? Similarly, if simultaneous entry into multiple markets is attempted, should signalling on these markets by a multimarket incumbent

[1] For a nice discussion of the welfare properties of such equilibria, see Tirole (1988).

[2] Saloner (1984) also finds that the welfare properties of equilibria in an alternative dynamic model of limit pricing are ambiguous.

[3] Several articles, such as Harrington (1986) and Jun and Park (2010), construct models in which the weak incumbent wants to price higher than the strong incumbent, either to signal to the entrant that it may have high costs or to encourage weak entrants to enter at the expense of stronger entrants.

be coordinated? If so, how? It quickly becomes clear that the full analysis of such markets goes well beyond the simple single incumbent/single entrant model.

A third industry in which the threat of entry can be identified clearly is the pharmaceuticals industry.[4] Producers of patented medicines are, by definition, not exposed to potential entry, but when patents expire, the possibility of entry and competition by producers of generic alternatives materializes. Because the expiry dates of patents is known, one may fruitfully analyze the pricing behavior of incumbent firms close to the expiration date of their patents and try to ascertain whether limit pricing is taking place. Interestingly, both advocacy groups and regulators have taken an active interest in this industry and its pricing behavior, but the academic literature on preexpiry pricing behavior is sparse. The Rx Price Watch Report (2011) from the AARP Public Policy Institute finds evidence that, for a number of blockbuster drugs, firms significantly *increase* their prices prior to patent expiry. This type of pricing behavior is noteworthy, because an ostensibly profit maximizing incumbent monopolist would *not* wish to increase its price in the face of entry. In fact, an increase in price is *prima facie* evidence that the firm was not charging its monopoly price till that point. This type of pricing turns out to be consistent with a type of equilibrium described in this article, namely, an equilibrium in which the informed firm sets an uninformative price for a while (i.e., pools) and then separates by setting a high monopoly price that subsequently prompts entry into the industry. Although suggestive, this phenomenon should be contrasted with another common practice which casts doubt on the limit pricing story, namely, the adoption of so-called pay-for-delay agreements. As noted by the Federal Trade Commission (2012), there is an increasing trend of incumbent firms engaging in this type of arrangement with potential generic competitors. Under a pay-for-delay agreement, the entrant delays its introduction of a competing generic drug, in return for the incumbent's promise not to introduce an *authorized generic* (i.e., a nonbranded version of the incumbent's drug that competes with the entrant's generic drug). To appreciate that this type of arrangement is incompatible with limit pricing, note that the incumbent firm is essentially forgoing future profits from the introduction of an authorized generic, in return for a short-term gain (i.e., that it will remain a monopolist in the short term). However, the basic mechanics of limit pricing are the exact opposite, namely, that the firm makes a short-term loss in order to secure a long-term gain.

To conclude, the extent to which limit pricing is helpful in explaining preexpiry pricing behavior in the pharmaceuticals industry is unclear.

Overall, the empirical literature does not allow one to confidently assert that limit pricing is a successful strategy for deterring entry (to the extent that the strategy is used at all, something that is also not clear). Being careful not to overstate the conclusions based on a theoretical analysis, the present article may suggest additional reasons why clear actual examples of limit pricing are hard to come by.

□ **The theoretical literature on limit pricing.** In his analysis, Bain (1949) identified two possible channels through which current prices may deter entry: (i) a low current price may signal to potential entrants that existing and future market conditions are unfavorable to entry, and that (ii) a low current price may signal to potential entrants something about the incumbent's response to entry. The first generation of contributions focused almost exclusively on explanation (ii) and featured models that were fully dynamic in nature, an approach which is suitable for the study of ongoing relationships between competing firms. This literature expounded a number of interesting characterizations of equilibrium limit price paths that could in principle be confronted with the data (see Carlton and Perloff, 2004). Nonetheless, most contributions had the unsatisfactory feature that entrants' decisions were not the outcome of rational deliberations, but rather mechanistic (assumed) responses to the incumbent's pricing behavior.[5] Furthermore, incumbents in these

[4] I thank an anonymous referee for directing my attention to this industry.

[5] Gaskins (1971) assumes that entry is a deterministic function of preentry price and Berck and Perloff (1988) that it is proportional to future profitability. Stochastic entry is assumed by Kamien and Schwartz (1971) and others.

models were endowed with perfect ability to commit to future (i.e., postentry) pricing behavior. This critique, first articulated by Friedman (1979), cast serious doubt on the received wisdom from Bain's insights. Milgrom and Roberts (1982) confronted this challenge by reformulating the situation as one of incomplete information and drew on both explanations (i) and (ii). In doing so, they succeeded in validating Bain's insights. Unlike the early theory on limit pricing, the Milgrom-Roberts analysis is essentially static in nature (because signalling through prices occurs once and for all and the potential entrant only has one opportunity to enter). This raises the question as to how robust their findings are to dynamic extensions and to what extent their predictions are reconcilable with those of the previous literature.

The second generation of models (distinguished by featuring incomplete information as initiated by Milgrom and Roberts), has greatly shaped the way economists think about limit pricing. As such, it is important to determine to what extent the lessons are robust to variations in the modelling approach. Matthews and Mirman (1983) consider the possibility that the incumbent's price provides only noisy information to the entrant about the profitability of entry. Under certain conditions, they find that limit pricing can be successfully employed by the incumbent to limit entry. Harrington (1986) considers a variation of the basic model in which the entrant is uncertain of his own costs, which are in turn correlated with those of the incumbent. This modelling approach means that a high preentry price may signal that the entrant's costs are likely to be high, thereby making entry less appealing. In turn, this may imply that in equilibrium, the incumbent charges a price higher than the monopoly price and also deters entry. Jun and Park (2010) consider a dynamic setup where the incumbent faces a sequence of entrants that can be either weak or strong, of which only the former can be deterred. Rather than having a strong opponent enter, the incumbent may wish to appear weak by charging a price higher than the monopoly price, thereby encouraging entry by weak entrants. This conclusion should be contrasted to that gained from the Milgrom-Roberts analysis.

The third generation of work on limit pricing seeks to come full circle by integrating the dynamic nature of first-generation models with a careful treatment of informational issues, as emphasized in second-generation models. The present article is a contribution to this branch of the literature. Closest to my analysis is the work of Kaya (2009), who studies repeated signalling in a reduced-form setup. She assumes one-sided asymmetric information and focuses on separating equilibria. Her work complements the current analysis, focusing on somewhat different issues. In particular, she does not select between equilibria and focuses on the least cost separating equilibrium, which allows the informed party to smooth costly signalling intertemporally.[6] In unpublished work, Saloner (1984) extends the Matthews and Mirman (1983) setup of noisy signalling to multiperiod settings. He assumes that not only is signalling noisy, but that market conditions evolve randomly over time. This introduces a real-options dimension to the entrant's entry problem, which may give it a strategic motive to delay entry. The evolving market approach is also taken by Roddie (2010), who treats signalling games with a particular monotone structure. A recent article by Gryglewicz (2009) treats a continuous-time signalling model in which the informed party's type is constant across time. His analysis focuses on pooling equilibria in which the incumbent's type is never revealed. Sorenson (2004) treats a dynamic model of limit pricing similar to the present one but implicitly assumes that the informed party is unable to credibly signal his type in a single period. This gives rise to repeated signalling over time.

In contrast to static models and to the existing dynamic models in the literature, my modelling approach allows me to study dynamic limit pricing behavior in which there is delayed revelation of information (i.e., separation may occur immediately, with a delay, or not at all). Furthermore, my analysis shows that once dynamic considerations are introduced, limit pricing may no longer be viable as an equilibrium phenomenon.

The remainder of the article is structured as follows. In Section 2, I introduce the benchmark static model that will constitute the building block of the dynamic analysis and then extend it to a

[6] This approach to repeated signalling was first adopted by Cho (1990).

fully dynamic model. A detailed analysis of the benchmark model is available as a web Appendix. I then analyze the dynamic setting and compare the outcomes of this analysis to the static setting. Furthermore, in Section 3, I perform comparative statics analysis and discuss sensitivity of equilibrium existence with respect to the length of the horizon and the discount factor. Section 4 concludes. Most proofs are relegated to the Appendix, whereas additional analysis and worked examples can be found in the web Appendix.

2. The model

■ In this section, I set out a dynamic model of limit pricing played between an incumbent firm and a potential entrant. The overall game consists of a number of rounds of pricing and entry decisions, with each round subdivided into three stages. For the sake of clarity, I will first describe a single round of the overall game.

□ **The benchmark setting.** Consider an incumbent monopolist I and a potential entrant E. The monopolist serves a market with demand $Q(p)$ and the entrant can enter the market at cost $F > 0$ to compete with the incumbent. The monopolist can be one of two types, high cost (H) or low cost (L), with probability μ and $(1 - \mu)$, respectively. The incumbent knows his type, but his type is unknown to the entrant (who only knows the probability μ). Let $C_H(q)$ and $C_L(q)$ be the cost functions of H and L, respectively. Denote by $\pi_i(p)$ the profit function of the incumbent of type $i = H, L$ when he sets price p. These profits are given by

$$\pi_i(p) = pQ(p) - C_i(Q(p)), \quad i = H, L. \tag{1}$$

Let D_i be the duopoly profit of the incumbent of type $i = H, L$ when competing against E and let $D_E(i)$ be the duopoly profits of E when competing against the incumbent of type $i = H, L$. Denote by p_H^M and p_L^M the monopoly prices under the technologies $C_H(\cdot)$ and $C_L(\cdot)$, respectively.

In the benchmark single-round setting, I make the following assumptions:

Assumptions.

1. $C_i(q)$, $i = H, L$ and $Q(p)$ are differentiable, for $q > 0$, and $p > 0$, respectively.
2. $C'_H(q) > C'_L(q), \forall q \in \mathbb{R}_+$, with $C_H(0) \geq C_L(0)$.
3. $Q'(p) < 0, \forall p \geq 0$.
4. $D_E(L) - F < 0$.
5. $D_E(H) - F > 0$.
6. $\pi_i(p)$ is strictly increasing for $p < p_i^M$ and strictly decreasing for $p > p_i^M$, $i = H, L$.
7. $\pi_i(p_i^M) > D_i, i = H, L$.
8. $\mu D_E(H) + (1 - \mu)D_E(L) - F < 0$.

Assumption 2 makes precise the sense in which type L is more efficient than type H. Assumption 3 simply states that demand is downward sloping. Assumptions 4–5 imply that E will not enter in the benchmark setting if he knows that I is of type L, whereas he will enter if he knows that I is of type H. Thus, these conditions are necessary for a separating limit price equilibrium to exist. Assumption 6 means that the incumbent's profit function is single peaked, whereas Assumption 7 ensures that entry deterrence is desirable for the incumbent, *ceteris paribus*. Under Assumption 8, the entrant expects to make negative profits against the incumbent in the benchmark setting if he cannot distinguish between the two types and thus stays out. This is a necessary condition for a pooling limit price equilibrium to exist. Once the game is extended to a multiple round interaction, Assumptions 4, 5, and 8 will have to be suitably modified, whereas the remaining assumptions will remain in place throughout.

Each round of the game between I and E is played in three stages.[7] At the first stage, I sets a price that will serve as a signal for E of I's type. After observing the price set by I, E decides at the second stage whether or not to enter (incurring the entry fee F). Denote E's entry decision by $s_E \in \{0, 1\}$, where $s_E = 0$ stands for *stay out* and $s_E = 1$ stands for *enter*. At the third stage, if E enters, he will learn I's type and compete against him in complete information fashion. Both incumbent and entrant discount the future by a factor $\delta \in [0, 1]$.

A strategy for I is a price for each of his two types, p_H or p_L, at the first stage, a price at the second stage if the entrant stays out, and a quantity or price to set at the third stage if the entrant enters (depending on the mode of competition), both as functions of his type and the decisions made at the first stage. A strategy for E is a decision rule to enter or not as a function of the price set by I at the first stage and a quantity or price to set at the third stage in case he enters (again, depending on the mode of competition).

In a single round, strategies are defined as follows. Let $\sigma \equiv (p_L, p_H, \overline{p})$ denote a triple of pure strategies of the game, that is, a price charged by each type of I and a threshold price governing E's entry decision. Throughout this article, attention will be restricted to pure strategy perfect Bayesian equilibria. Denote by p_H^* and p_L^* the equilibrium prices charged by the H type, and the L type, respectively.

Definition 1. The triple σ is a separating equilibrium if $p_H^* \neq p_L^*$ and a pooling equilibrium if $p_H^* = p_L^*$. Furthermore, σ is a limit price equilibrium if $p_H^* < p_H^M$ or $p_L^* < p_L^M$ or both.

Note that under the maintained assumptions, *ceteris paribus*, the high-cost incumbent will wish to set a higher monopoly price than the low-cost incumbent. An implication of this fact is that an inefficient incumbent would only set lower prices than an inefficient incumbent's monopoly price, in order to convince the entrant that it is in fact an efficient incumbent.

Summary of the benchmark setting. The analysis of the single-round model is well understood and the details are therefore omitted (see the web Appendix for a complete analysis with the present notation). If the game admits equilibria, there are typically a continuum of such equilibria within each class, that is, a continuum of separating equilibria and a continuum of pooling equilibria. The multiplicity relies on choosing different beliefs off the equilibrium path. Using standard equilibrium selection techniques, such as equilibrium dominance, a unique equilibrium can be selected within each class.

In addition, after performing equilibrium selection, the set of equilibria can, if nonempty, be divided into two distinct regimes, namely, a *limit price regime* and a *monopoly price regime*. These regimes will reappear in an important way in the dynamic game. In the monopoly price regime, the unique equilibrium satisfying equilibrium dominance is characterized by firms separating by setting their respective monopoly prices, whereas in the limit price regime, both pooling and separating limit price equilibria coexist, both satisfying equilibrium dominance. Which regime obtains, depends on the parameter constellation and on the specifics of the mode of competition.

For later reference, the monopoly price regime obtains if and only if

$$\pi_H(p_H^M) + \frac{\delta D_H}{1-\delta} \geq \frac{\pi_H(p_L^M)}{1-\delta}. \tag{2}$$

This inequality has an interesting interpretation. The left-hand side is the profit for the H type of revealing his type by earning monopoly profits in the first period and then earning discounted duopoly profits in perpetuity thereafter. The right-hand side is the discounted profit stream for the H type from mimicking the L type's monopoly price in perpetuity.

[7] In the dynamic version of the model, stages one and two will together constitute a period and stage three will be a separate period.

□ **The dynamic setting.** To introduce proper dynamics to the model, the benchmark model is repeated $T - 1$ times as long as entry has not occurred (so that period $T < \infty$ is the last period and period $T - 1$ is the last period in which signalling and/or entry may occur).[8] Note that this is *not* a repeated game, as entry can only occur once and thus the stage game is not unvarying across periods; hence the use of the term *rounds*.

Next, I formally define what is meant by a separating and a pooling equilibrium in this dynamic setting. Let $\sigma^T \equiv \{p_{t,L}, p_{t,H}, \overline{p}_t\}_{t=1}^{T-1}$ denote a triple of pure strategies of the game. Denote by $\left\{p^*_{t,L}\right\}_{t=1}^{T-1}$ and $\left\{p^*_{t,H}\right\}_{t=1}^{T-1}$ the equilibrium price sequences for the two types of incumbent.

Definition 2. The triple σ^T is a separating equilibrium if $\left\{p^*_{t,H}\right\}_{t=1}^{T-1} \neq \left\{p^*_{t,L}\right\}_{t=1}^{T-1}$ and a pooling equilibrium if $\left\{p^*_{t,H}\right\}_{t=1}^{T-1} = \left\{p^*_{t,L}\right\}_{t=1}^{T-1}$.

These definitions are the natural generalizations of their static counterparts. In essence, they extend the notion that upon observing the incumbent's equilibrium strategy, the entrant can infer the incumbent's type. Importantly though, it is quite possible that such an inference can only be made upon observing the *entire* strategy. One reason for adopting this definition is that if the incumbent's type has to be recognizable after all partial (i.e., nonterminal) histories, as is the case in Kaya (2009) and Noldeke and van Damme (1990), then there cannot *by assumption* be any delay in separation. I shall not impose such a restriction, as it may rule out interesting equilibria with delayed information revelation. In what follows, it is useful to distinguish between *immediate separation equilibria* and *delayed separation equilibria* (of which a special case is the pooling equilibrium).

As in any signalling game, out-of-equilibrium beliefs must be assigned. An optimal decision rule for the entrant will prescribe entry if the incumbent is believed to be of the H type and no entry otherwise, that is, if either the incumbent is believed to be of the L type or the two types cannot be distinguished.[9] I will assume for simplicity that the incumbent will be interpreted to be of the H type for any observed price above the L type's equilibrium price (either separating or pooling) and to be the L type, otherwise. These beliefs amount to the following (optimal) monotone decision rule as a function of the observed price p_t for $t = 1, \dots, T - 1$, if entry has not occurred by time t:

$$s_E(p_t) = \begin{cases} 1 & \text{if} \quad p_t > \overline{p}_t \\ 0 & \text{if} \quad p_t \leq \overline{p}_t, \end{cases} \tag{3}$$

for an appropriately chosen sequence $\left\{\overline{p}_t\right\}_{t=1}^{T-1}$ (determined by E).[10] Note that although the dependence of the decision rule on past price observations has been suppressed in the notation, the entrant is allowed to condition his entry decision on all available information, including the exact history of prices set by the incumbent.

□ **Equilibrium selection in the dynamic model.** Rather than characterizing the entire set of equilibria of the dynamic game, I will argue that only a subset of the equilibria are reasonable in a sense to be made precise.[11] Note that *ex post* separation, the sequence $(p_L^M, p_L^M, ..., p_L^M)$, yields the *highest possible* payoff to the L type incumbent (as long as the entrant's beliefs that he is facing the L type are not disturbed). In other words, as long as the L type sticks to this price

[8] Although the benchmark model has two periods, it is static in the sense that signalling and entry can take place only once.

[9] Assumptions on the primitives of the model that ensure the optimality of this decision rule will be introduced below.

[10] This type of monotone strategy is similar to the trigger strategies considered by Saloner (1984).

[11] A more complete discussion of this concept and its relation to the existing literature can be found in the web Appendix.

sequence, there is no possible deviation that can yield a higher payoff to him for *any* beliefs that a deviation could feasibly induce. On the other hand, there are deviations that would make the H type strictly better off. For example, consider the sequence $(p_H^M, p_L^M, ..., p_L^M)$. If the entrant gives the incumbent the benefit of the doubt and ignores the out-of-equilibrium price p_H^M (which he is entitled to do because there are no restrictions on beliefs off the equilibrium path), then the H type is strictly better off under this price sequence than under the sequence $(p_L^M, p_L^M, ..., p_L^M)$ in which he mimics the strategy of the L type incumbent. However, this means that the set of best responses of the entrant that makes the H type want to deviate is strictly larger than the set of best responses that would induce the L type to deviate (because this latter set is empty). This line of reasoning implies that the entrant should actually conclude that it was the high-cost incumbent who deviated, thus justifying the removal of this potential Bayesian Nash equilibrium. This is very similar to the heuristic embodied in criterion D1. The reason that this is not simply an application of the standard D1 refinement is that the deviation is compared to the equilibrium under complete information rather than to an arbitrary equilibrium. In what follows, the analysis will be confined to equilibria that are selected using this approach. This anchored D1 criterion is used in all postseparation periods. In the separation period itself, where beliefs are not degenerate, I will apply standard equilibrium dominance to select equilibria.[12]

This approach to equilibrium selection may have applicability to a larger class of dynamic signalling models. For that reason, I will now formally define the anchored D1 criterion. The D1 criterion is usually defined for static signalling games, typically by making use of reduced-form payoff functions for the sender and the receiver (i.e., the informed and the uninformed party, respectively). In what follows, the reduced-form payoff functions will be composed of two separate parts, namely, the payoff in the current period and the discounted expected payoffs from future equilibrium play. Using continuation equilibrium play in this way allows one to make use of the D1 criterion period by period, as in Roddie (2010) and Gedge, Roberts, and Sweeting (2013).

First, define the reduced-form payoff to the type $i = H, L$ incumbent as

$$\Pi_i^t(p, s_E) \equiv \pi_{t,i}(p) + \delta V_i^t(p, s_E, \mu'). \tag{4}$$

In this definition, $V_i^t(p, s_E, \mu')$ is the expected equilibrium continuation value (for some given equilibrium) for the incumbent when setting price p in the current period, the entrant's entry decision is s_E, and the entrant's beliefs upon observing p are given by μ'.

For any period $t = 1, ..., T - 1$ for which beliefs are nondegenerate, the usual equilibrium dominance criterion is applied. To define D1, let S be a nonempty subset of the type space $\{H, L\}$ and define the entrant's best response

$$BR(S, p) \equiv \cup_{\mu':\mu(S|p)=1} BR(\mu', p), \tag{5}$$

where

$$BR(\mu', p) \equiv \arg \max_{s_E \in \{0,1\}} \sum_{i \in \{H,L\}} \mu(i|p)\Pi_E(p, s_E, i) \tag{6}$$

and $\mu(i|p)$ is the entrant's belief assigned to type i upon observing price p set by the incumbent. The term $\Pi_E(p, s_E, i)$ is simply the entrant's discounted, expected payoff in the given equilibrium. Pick an equilibrium in which the type i incumbent's payoff is Π_i^* and define the sets

$$\mathcal{D}(i, S, p) \equiv \cup_{\mu:\mu(S|p)=1} \left\{p_E \in BR(\mu, p) : \Pi_i^* < \Pi_i^t(p, s_E)\right\} \tag{7}$$

$$\mathcal{D}^0(i, S, p) \equiv \cup_{\mu:\mu(S|p)=1} \left\{p_E \in BR(\mu, p) : \Pi_i^* = \Pi_i^t(p, s_E)\right\}. \tag{8}$$

[12] An alternative to this approach would be to employ the standard D1 criterion to preseparation periods. This would effectively rule out delayed-entry equilibria.

The set $\mathcal{D}(i, S, p)$ is simply the best responses for the entrant that make the incumbent want to deviate from the equilibrium action, whereas $\mathcal{D}^0(i, S, p)$ is the set of best responses for which the incumbent is indifferent between deviating and taking the equilibrium action.

The standard definition of D1 (in the formulation of Fudenberg and Tirole, 1991) is then as follows:

Prune the type-strategy pair (i, p) *under criterion D1 if there exists some type* $j \in \{H, L\}$ *such that*

$$\left\{\mathcal{D}(i, \{H, L\}, p) \cup \mathcal{D}^0(i, \{H, L\}, p)\right\} \subset \mathcal{D}(j, \{H, L\}, p). \tag{9}$$

This definition means that if the set of entry decisions for the entrant that makes type $i = H, L$ willing to deviate to some price p is strictly smaller than the set of of entry decisions that makes type j willing to deviate, then the entrant should believe it to be infinitely more likely that the deviation to price p came from type j rather than from type i.

Next, for any period $t = 1, ..., T-1$ for which beliefs are degenerate, apply criterion D1 as above but replacing the equilibrium payoff Π_i^* to the incumbent by $\overline{\Pi}_i^*$, which is the discounted equilibrium payoff to the incumbent under complete information (given the degenerate beliefs).

In what follows, I will make use of the following definition:

Definition 3. An equilibrium price sequence $\left\{p_{t,i}^*\right\}_{t=1}^{T-1}$, $i = H, L$ satisfies the anchored D1 criterion, if the anchored D1 criterion is applied separately to each preentry period in which beliefs are degenerate.

In the present model, once beliefs are centered on the H type, entry occurs and no further learning can take place (because the incumbent's type is then perfectly revealed). In more general settings, further rounds of equilibrium selection may be necessary and thus, somewhat more delicate arguments would be needed to select between equilibria.

□ **Separating limit price equilibria.** To make limit pricing with separation feasible, it must be the case that the entrant would find it optimal to enter against the H type incumbent but to stay out against the L type incumbent. A necessary condition for a separating limit price equilibrium with separation in any period $t = 1, ..., T-1$ to exist is that

$$D_E(L) < \left(\frac{1-\delta}{1-\delta^{T-t+1}}\right) F < D_E(H), \quad t = 1, ..., T. \tag{10}$$

As the coefficient on the entry fee F in this condition is decreasing in the number of remaining periods, the condition may fail to hold for some t.[13] In order to avoid time-varying necessary conditions at this stage of the analysis, I instead impose the following restrictions, which ensure that separation is feasible in an arbitrary period $t = 1, ..., T$:

Assumptions.

4′. $D_E(L) < (\frac{1-\delta}{1-\delta^T})F$.
5′. $D_E(H) > F$.

It should be pointed out that although Assumption A5' coincides with Assumption A5, Assumption A4' is stronger than Assumption 4. In Section 3, I will explicitly analyze the dependence of these conditions on the discount factor and the length of the interaction.

[13] In particular, it may be the case that the necessary condition for a separating limit price equilibrium to be feasible is that the remaining number of periods be small. This case will be considered in the next section.

Characterization. The characterization of equilibria of the dynamic model follows similar steps as that of the single-round model, although the analysis is complicated by the dynamic nature of the problem. Based on the discussion above, a separating equilibrium price sequence $\{p^*_{t,L}\}_{t=1}^{T-1}$ for the L type satisfying the anchored D1 refinement is of the general form $(p^*_1, ..., p^*_{t-1}, p^*_{t,L}, p^M_L, ..., p^M_L)$, with separation occurring in period $t = 1, ..., T-1 \leq \infty$.[14]

Next, the entrant's strategy can be characterized as follows:

Lemma 1 (entrant's optimal decisions). Consider the equilibrium price sequence $(p^*_1, ..., p^*_{t-1}, p^*_{t,L}, p^M_L, ..., p^M_L)$ in which separation occurs in period $t = 1, ..., T-1$. Then, (i) $\overline{p}_s = p^*_s$ and $\overline{p}_s \leq p^M_L$, $\quad s = 1, ..., t-1$, (ii) $\overline{p}_t = p^*_{t,L}$ and $\overline{p}_t < p^M_L$, and (iii) $\overline{p}_s = p^*_{s,L} = p^M_L$, $\quad s = t+1, ..., T-1$.

Proof. The proofs of (i) and (ii) parallel those in the single-round setting (see the web Appendix) and are omitted, whereas that of (iii) follows from the equilibrium selection approach discussed above. ■

It should be emphasized that I do not make any use of support restrictions in the postseparation game. With a support restriction and assuming that prices have revealed that the incumbent is of type L, the two different sequences of postseparation prices $(p^M_L, p^M_L, p^M_L, p^M_H, ...)$ and $(p^M_L, p^M_L, p^M_L, p^M_L, ...)$ would be treated equivalently in terms of beliefs and entry decisions, whereas with the equilibrium selection procedure I use, the former sequence would prompt the entrant to update his beliefs and subsequently enter.

I now proceed by first analyzing the incentive compatibility constraints of each type of incumbent and then move on to the issues of equilibrium existence and selection. Although matters are complicated somewhat by the dynamic nature of the model (there are in each case two regimes to consider, which depend on parameter values), the basic progression of the analysis is straightforward.

The incentive compatibility constraints. As is the case in the single-round setting, the best alternative for the L type to setting the separating equilibrium price is to set his monopoly price. In contrast, the best alternative for the H type to setting the separating equilibrium price, that is, his monopoly price, is to mimic the L type's equilibrium price. With this in mind, the following partial characterization of the separating equilibrium price can be given, using the L type's incentive compatibility constraints:

Lemma 2 (efficient incumbent's incentive constraints). For the price sequence $(p^*_1, ..., p^*_{t-1}, p^*_{t,L}, p^M_L, ..., p^M_L)$ to constitute a separating limit price equilibrium, it must satisfy

$$\pi_L(p^*_s) \geq (1-\delta)\pi_L(p^M_L) + \delta D_L, \quad p^*_s < p^M_L, \quad s = 1, ..., t-1 \tag{11}$$

$$\pi_L(p^*_{t,L}) \geq \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p^M_L) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_L. \tag{12}$$

Proof. See the Appendix. ■

[14] In fact, in a separating equilibrium satisfying the anchored D1 criterion, it must also be the case that $p^*_s = p^M_L$ for $s = 1, ..., t-1$, as will be shown below.

To write the incentive compatibility constraint for the separation period in terms of prices, define the following set:

$$A_L(T,t) \equiv \left\{ p : \pi_L(p) = \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_L \right\}. \tag{13}$$

As $\pi_L(p) = D_L$ for some p, then by Assumptions 6 and 7, it follows that the set $A_L(T,t)$ is nonempty and contains at most two points. Let

$$\alpha_0(T,t) \equiv \min A_L(T,t), \quad \beta_0(T,t) \equiv \max A_L(T,t), \tag{14}$$

where $\alpha_0(T,t) < \infty$ and $\beta_0(T,t) \leq \infty$. Let the single-round cutoffs be denoted by α_0 and β_0, that is, $\alpha_0 \equiv \alpha_0(2,1)$ and $\beta_0 \equiv \beta_0(2,1)$.

In terms of prices, the L type's incentive compatibility constraint for the separation period can then be written as

$$p_{t,L}^* \in [\alpha_0(T,t), \beta_0(T,t)]. \tag{15}$$

For later use, note that by definition, it is the case that

$$\pi_L(\alpha_0(T,t)) = \left(1 - \frac{\delta - \delta^{T-t+1}}{1-\delta}\right)\pi_L(p_L^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_L = \pi_L(\beta_0(T,t)). \tag{16}$$

I next consider the incentive compatibility constraints for the H type. These are slightly more complicated than those of the L type, due to the fact that the H type may, in general, wish to mimic the behavior of the L type for an arbitrary number of periods after the L type has chosen to separate. To see this more clearly, consider an equilibrium price sequence for the L type given by $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ for $t = 1, ..., T-1$. In equilibrium, the H type's strategy is given by a sequence $(p_1^*, ..., p_{t-1}^*, p_H^M, x_{t+1}, ..., x_T)$ where x_s is shorthand for H's postentry equilibrium strategy in period $s = t+1, ..., T$.

Consider possible deviations for the H type. First, H may wish to deviate during periods with pooling, and so these pooling prices must respect appropriate incentive compatibility constraints. Next, the H type incumbent may deviate in the period where separation is prescribed by mimicking the L type's strategy. Last, H may deviate by not only mimicking the L type's separating price, but also by mimicking L's postseparation strategy p_L^M for an arbitrary number of periods. It turns out that the optimal amount of mimicking undertaken by the H type out of equilibrium depends in a simple way on parameter values, as the following results show:

Lemma 3 (inefficient incumbent's incentive constraints).

(i) In the monopoly price regime, mimicking only once is the optimal off-equilibrium path strategy. Furthermore, for the price sequence $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ for $t = 1, ..., T-1$ to constitute a separating limit price equilibrium, it must satisfy

$$\pi_H(p_s^*) \geq (1-\delta)\pi_H(p_H^M) + \delta D_H, \quad p_s^* < p_L^M, \quad s = 1, ..., t-1 \tag{17}$$

$$\pi_H(p_{t,L}^*) \leq (1-\delta)\pi_H(p_H^M) + \delta D_H. \tag{18}$$

(ii) In the limit price regime, mimicking perpetually is the optimal off-equilibrium path strategy. Furthermore, for the price sequence $(p_1^*, ..., p_{t-1}^*, p_{t,L}^*, p_L^M, ..., p_L^M)$ for $t = 1, ..., T-1$ to constitute a separating limit price equilibrium, it must satisfy

$$\pi_H(p_s^*) \geq (1-\delta)\pi_H(p_H^M) + \delta D_H, \quad p_s^* < p_L^M, \quad s = 1, ..., t-1 \tag{19}$$

$$\pi_H(p_{t,L}^*) \leq (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-t+1}}{1-\delta}\right) D_H - \left(\frac{\delta - \delta^{T-t}}{1-\delta}\right)\pi_H(p_L^M). \tag{20}$$

Proof. See the Appendix. ■

To express these incentive compatibility constraints in terms of prices, define the following set:

$$A_H(T,t) \equiv \left\{ p : \pi_H(p) = (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta-\delta^{T-t+1}}{1-\delta}\right) D_H - \left(\frac{\delta-\delta^{T-t}}{1-\delta}\right)\pi_H(p_L^M) \right\}. \tag{21}$$

Note that the coefficients on $\pi_H(p_H^M)$, D_H, and $\pi_H(p_L^M)$ in the definition of $A_H(T,t)$ sum to one. It then follows from Assumptions 6 and 7 and the fact that $\pi_H(p_H^M) > \pi_H(p_L^M)$ that the set $A_H(T,t)$ contains at most two points. Let

$$\widehat{\alpha}(T,t) \equiv \min A_H(T,t), \quad \widehat{\beta}(T,t) \equiv \max A_H(T,t), \tag{22}$$

where $\widehat{\alpha}(T,t) < \infty$ and $\widehat{\beta}(T,t) \le \infty$. Let the single-round cutoffs be denoted by $\widehat{\alpha}$ and $\widehat{\beta}$, that is, $\widehat{\alpha} \equiv \widehat{\alpha}(2,1)$ and $\widehat{\beta} \equiv \widehat{\beta}(2,1)$.

For periods with pooling, the price sequence must thus satisfy

$$\max\{\alpha_0, \widehat{\alpha}\} \le p_s^* \le p_L^M < p_H^M, \quad s = 1, ..., t-1. \tag{23}$$

For the period in which separation is prescribed, the H type's incentive compatibility constraint when condition (2) is satisfied is that

$$p_{t,L}^* \notin [\widehat{\alpha}, \widehat{\beta}], \tag{24}$$

which is as in the single-round setting. In this case, only the inequality $p_{t,L}^* \le \widehat{\alpha}$ is relevant, because $p_{t,L}^* < p_L^M < p_H^M < \widehat{\beta}$.

For the period in which separation is prescribed, the H type's incentive compatibility constraint when condition (2) is violated is that

$$p_{t,L}^* \notin [\widehat{\alpha}(T,t), \widehat{\beta}(T,t)]. \tag{25}$$

In this case, only the inequality $p_{t,L}^* \le \widehat{\alpha}(T,t)$ is relevant, because $p_{t,L}^* < p_L^M < p_H^M < \widehat{\beta}(T,t)$. For later use, note that by definition, it is the case that

$$\pi_H(\widehat{\alpha}(T,t)) = (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta-\delta^{T-t+1}}{1-\delta}\right) D_H - \left(\frac{\delta-\delta^{T-t}}{1-\delta}\right)\pi_H(p_L^M) = \pi_H(\widehat{\beta}(T,t)). \tag{26}$$

Before summarizing the analysis of the dynamic limit price equilibria, I will briefly discuss the issue of equilibrium existence.

Existence of separating limit price equilibria. When (2) is satisfied, the existence of separating limit price equilibria is ensured if $\widehat{\alpha} > \alpha_0(T,t)$, whereas if (2) is violated, then existence is ensured if $\widehat{\alpha}(T,t) > \alpha_0(T,t)$.

The relevant sufficient conditions for the existence of separating limit price equilibria are as follows:

Proposition 1 (existence of separating limit price equilibria).

(i) In the monopoly price regime, if

$$\pi_L(p_L^M) - D_L > \left(\frac{1-\delta}{1-\delta^{T-t}}\right)\left[\pi_H(p_H^M) - D_H\right], \quad t = 1, ..., T-1, \tag{27}$$

then $\widehat{\alpha} > \alpha_0(T,t)$ and the set of separating limit price equilibria is nonempty.

(ii) In the limit price regime, if

$$\pi_L(p_L^M) - D_L > \left[\frac{(1-\delta)\delta^{T-t-1}}{1-\delta^{T-t}}\right]\pi_H(p_H^M) - D_H + \left[\frac{\delta - \delta^{T-2}}{1-\delta^{T-t}}\right]\pi_H(p_L^M),$$
$$t = 1, ..., T-1, \tag{28}$$

then $\widehat{\alpha}(T,t) > \alpha_0(T,t)$ and the set of separating limit price equilibria is nonempty.

Proof. See the Appendix. ■

Note that in both regimes, the relevant sufficient condition for existence becomes easier to satisfy as the horizon recedes, that is, equilibrium may exist in the dynamic setting even if none exist in the static setting.

Equilibrium selection. I now determine which of the equilibria in the dynamic game satisfy the anchored D1 criterion after separation has taken place. I do this explicitly for the case where (2) is violated. The case where (2) is satisfied follows similar steps, with $\widehat{\alpha}(T,t)$ replaced by $\widehat{\alpha}$.

Proposition 2 (uniqueness of separating limit price equilibrium satisfying anchored D1).

(i) In the limit price regime, only $p_{t,L}^* = \widehat{\alpha}(T,t)$ satisfies the anchored D1 criterion.
(ii) In the monopoly price regime, only $p_{t,L}^* = p_L^M$ satisfies the anchored D1 criterion.

Proof. See the Appendix. ■

Before turning to the comparative statics analysis, the following result is shown:

Proposition 3 (characterization of separating limit price equilibrium satisfying anchored D1).

(i) In the monopoly price regime, all equilibria satisfying the anchored D1 refinement are immediate separation equilibria.
(ii) In the limit price regime, all equilibria satisfying the anchored D1 refinement are of a form where, for $t = 1, ..., T-1$, the L type's strategy is given by a sequence $(p_L^M, ..., p_L^M, p_{t,L}^*, p_L^M, ..., p_L^M)$ and the H type's strategy is given by a sequence $(p_L^M, ..., p_L^M, p_H^M, x_{t+1}, ..., x_T)$, where x_s is the H type's postentry strategy at time $s = t+1, ..., T$.

Proof. (i) It can be shown in the single-round setting that in the monopoly price regime, that is, when (2) is satisfied, no pooling equilibria surviving equilibrium dominance exist (see the web Appendix). The result then follows immediately from observing that the incentive compatibility constraint of the H type in periods of preseparation pooling are identical to the incentive compatibility constraint in the single-round setting. (ii) The proof follows directly from the lemmas proved above. ■

This result means that in the monopoly price regime, the unique prediction is that each type of incumbent will set its corresponding monopoly price in the first period and deter entry in case of type L and invite entry in case of type H. That is, the equilibrium is necessarily an immediate separation equilibrium. In the limit price regime, equilibria are possibly of the delayed separation variety.

□ **Pooling limit price equilibria.** In the dynamic setting, a pooling equilibrium consists of a price sequence $\sigma^T = \{p_t^*\}_{t=1}^{T-1}$ set by both types of incumbent. This means that in every period, the entrant cannot distinguish the two types. For pooling to be feasible in period $t = 1, ..., T-1$, the following conditions need to be imposed:

$$\mu D_E(H) + (1-\mu)D_E(L) < \left(\frac{1-\delta}{1-\delta^{T-t+1}}\right)F, \quad t = 1, ..., T. \tag{29}$$

These constraints are more difficult to satisfy the farther away the final period is. In order to avoid time-varying necessary conditions at this stage of the analysis, I instead impose the following condition that ensures that pooling is feasible in any arbitrary period $t = 1, ..., T-1$:

Assumption 8'. $\mu D_E(H) + (1-\mu)D_E(L) - (1-\delta)F < 0.$

Interestingly, this condition is more difficult to satisfy than that in Assumption 8, pertaining to the benchmark setting. In other words, once dynamics are introduced, the necessary condition for a pooling equilibrium to be feasible is more difficult to satisfy. Furthermore, it becomes increasingly difficult the more patient the entrant becomes, that is, the larger the discount factor δ becomes. This feature will be further explored in the next section.

The following characterization of the entrant's decision rule holds:

Lemma 4 (entrant's optimal decisions).
$\overline{p}_t = p_t^*$ and $p_t^* \leq p_L^M, \quad t = 1, ...T-1.$

Proof. The proof is omitted. ■

The incentive compatibility constraints. As is the case in the single-round setting, the best alternative for each type to setting the pooling price is to set the monopoly price and thus invite entry. With this in mind, the following can be shown to hold:

Lemma 5 (incumbent's incentive constraints). For the price sequence $\left\{p_t^*\right\}_{t=1}^{T-1}$ to constitute a pooling limit price equilibrium, it must satisfy

$$\pi_L(p_t^*) \geq (1-\delta)\pi(p_L^M) + \delta D_L, \quad p_t^* < p_L^M, \quad t = 1, ..., T-1 \tag{30}$$

$$\pi_H(p_t^*) \geq (1-\delta)\pi(p_H^M) + \delta D_H, \quad t = 1, ..., T-1. \tag{31}$$

Proof. See the Appendix. ■

These results can be collected as follows:

Proposition 4 (characterization of pooling limit price equilibria). In any pooling limit price equilibrium, it must be the case that

$$\max\{\alpha_0, \widehat{\alpha}\} \leq p_t^* \leq p_L^M < p_H^M, \quad t = 1, ..., T-1. \tag{32}$$

Equilibrium selection. The incentive compatibility constraints in the dynamic pooling equilibrium are in fact equivalent to their static counterparts. It then follows from the same arguments as in the static analysis that only $p_t^* = p_L^M$ satisfies equilibrium dominance (see the web Appendix for details).

□ **Existence of limit price equilibria satisfying anchored D1 criterion.** As is the case in the analysis of the single-round setting, one may characterize two distinct regimes, namely, a monopoly price regime and a limit price regime. In the monopoly price regime, the only outcome consistent with the anchored D1 refinement is separation on monopoly prices in the first period, whereas in the limit price regime, both pooling and separating equilibria coexist, both satisfying the anchored D1 refinement. In the pooling equilibrium, both types of incumbent set the efficient type's monopoly price and thus, the equilibrium involves limit pricing. In the separating limit price equilibrium, however, because the benefits from entry deterrence increase with the horizon and the patience of the players, credibly signalling to be of the efficient type may involve incurring arbitrarily large losses in the period in which separation is prescribed. Depending on the model specification and mode of competition in the market game, this may actually involve setting negative prices.[15]

The equilibrium price paths of the dynamic model should be contrasted to those of the early limit pricing literature. As Carlton and Perloff (2004) nicely show, some models predict that equilibrium prices will increase over time, others that they will decrease, and yet others that price paths are not necessarily monotone. Because of the relatively weak restrictions on equilibrium behavior imposed by the incentive compatibility constraints, many different price profiles can be sustained in equilibrium. However, not all such profiles are consistent with the refinements used in the present analysis.

In the *monopoly price regime*, the analysis predicts immediate separation on monopoly prices, with resulting entry against the H type incumbent and no entry against the L type incumbent (who will subsequently charge monopoly prices indefinitely). In the *limit price regime*, all equilibria share the same overall structure. Namely, they are characterized by a nonnegative and possibly infinite number $N = 0, 1, \ldots$ of periods in which the two types of incumbent pool on the efficient type's monopoly price p_L^M, followed by a period $N + 1$ in which the firms separate. In case the incumbent is of type L, prices will dip in order to signal strength, after which prices will return to the preseparation level p_L^M. In case the incumbent is of type H, prices will jump to p_H^M and then fall to some level $p < p_H^M$ (because of the ensuing entry and competition that will drive down prices).

Interestingly, in this model, the timing of separation is indeterminate in the sense that in equilibrium, signalling can happen in any period, if ever. In other words, equilibrium does not pin down if and when signalling will take place. Note that this result is entirely unrelated to the equilibrium multiplicity created by choosing different off-equilibrium path beliefs in usual signalling games. Instead, the multiplicity is related to the coexistence of different classes of equilibria, that is, pooling and separating equilibria. In the static benchmark setting, if both types of equilibria exist, there is no way to determine which of such different equilibria will be played. A similar situation arises in the dynamic setting, where separation may be preceded by multiple rounds of pooling. This indeterminacy effectively means that there are multiple equilibria (among which we cannot select) which differ in their predictions on the timing of separation and possible entry. It should be emphasized that timing indeterminacy is not inherently because of the dynamics of the model. Even in a static framework in which pooling and separating equilibria coexist, there is indeterminacy in this sense.[16]

Note that both types of incumbent are better off the later separation occurs. The efficient type earns monopoly profits as long as entry does not occur and is not called upon to engage in costly signalling. In turn, the inefficient type effectively deters entry as long as pooling takes place. Although pooling is indeed costly for the inefficient type, it still dominates entry. Therefore, it is not possible to use separation date as a screening device.

[15] This will be the case, for example, in a model with constant marginal costs and linear demand as that considered by Tirole (1988). In fact, the efficient incumbent would have to give its customers infinitely large subsidies to credibly convey his identity.

[16] One can then think of the separating equilibrium as an immediate separation equilibrium and of the pooling equilibrium as a delayed separation equilibrium.

3. Comparative analysis

■ To fully explore the differences between the static and dynamic settings, I will now consider the effects of changing the main distinguishing features of the dynamic setting, namely, the length of the interaction T and the discount factor δ. I will in turn analyze the effects on the necessary conditions for the entrant and the incumbent, respectively. For the former, the relevant conditions are A4'–A5', which ensure that E wishes to enter against H and to stay out against L and (29) (or A8'), which ensures that an uninformed entrant wishes to stay out. For the latter, the relevant conditions are the incentive compatibility constraints (12) and (20). Last, I will also analyze the effects on the necessary condition for a separating equilibrium to exist.

It is immediately clear that the constraints in preseparation periods are unaffected by the length of the horizon. In periods where separation is prescribed, however, the constraints do explicitly depend on the remaining number of periods (if $T < \infty$). First, consider the L type's incentive compatibility constraint $p^*_{t,L} \geq \alpha_0(T,t)$. The cutoff $\alpha_0(T,t)$ is decreasing in T as $\alpha_0(T,t) \leq p^M_L$ and the right-hand side of the equality defining the set $A_L(T,t)$ is decreasing in T. In the limit $T \to \infty$, $\alpha_0(T,t)$ is implicitly given by

$$\lim_{T\to\infty} \pi_L(\alpha_0(T,t)) = \left(1 - \frac{\delta}{1-\delta}\right)\pi_L(p^M_L) + \left(\frac{\delta}{1-\delta}\right)D_L. \tag{33}$$

This means that as the horizon recedes, the L type's incentive compatibility constraint becomes easier to satisfy.

Now, turn to the H type. In the monopoly price regime, the H type's incentive compatibility constraints are unaffected by changes in T. In the limit price regime, however, the appropriate constraint is $p^*_{t,L} \leq \widehat{\alpha}(T,t)$. The cutoff $\widehat{\alpha}(T,t)$ is decreasing in T as $\widehat{\alpha}(T,t) \leq p^M_H$ and the right-hand side of the equality defining the set $A_H(T,t)$ is decreasing in T. In the limit $T \to \infty$, $\widehat{\alpha}(T,t)$ is implicitly given by

$$\lim_{T\to\infty} \pi_H(\widehat{\alpha}(T,t)) = \pi_H(p^M_H) + \left(\frac{\delta}{1-\delta}\right)\left(D_H - \pi_H(p^M_L)\right). \tag{34}$$

As the horizon recedes, the H type's incentive compatibility constraint becomes more difficult to satisfy.

These observations mean that, in the monopoly price regime, the set of separating limit price equilibria expands with the length of the horizon, but the only equilibrium satisfying the anchored D1 refinement remains unchanged, namely, immediate separation on monopoly prices. In the limit price regime, both critical cutoffs $\widehat{\alpha}(T,t)$ and $\alpha_0(T,t)$ decrease in the length of the horizon T, and so both the largest and the smallest separating (and limiting) equilibrium prices decrease. Although the effect of an increase in T on the set of equilibrium prices is ambiguous, the unique equilibrium limit price satisfying the anchored D1 refinement is unambiguously decreasing. I gather these results in the following proposition (the following results implicitly assume that the relevant entry constraints A4' and A5' are satisfied for the entrant):

Proposition 5 (dependence of equilibrium on length of interaction).

(i) In the monopoly price regime, the unique equilibrium price satisfying the anchored D1 refinement is invariant in the length of the interaction.
(ii) In the limit price regime, the unique separating equilibrium limit price satisfying the anchored D1 refinement is decreasing in the length of the interaction.

□ **Effects on the cost of signalling.** In this subsection, I consider how the incumbent's cost of signalling changes when dynamics are introduced. In the single-round setting, the H type's profits from mimicking the L type's separating equilibrium strategy may be positive. Interestingly, this is no longer necessarily the case in the dynamic version of the game. In particular, I have the following result:

Proposition 6 (cost of signalling with infinite horizon). In the limit price regime,

$$\lim_{T\to\infty} \pi_H(\widehat{\alpha}(T,t)) < 0. \tag{35}$$

Proof. See the Appendix. ■

In fact, the result becomes even stronger as the future becomes increasingly important, as the next result demonstrates:

Corollary 1 (cost of signalling with high discount factor). In the limit price regime,

$$\lim_{\delta\to 1}\lim_{T\to\infty} \pi_H(\widehat{\alpha}(T,t)) = -\infty.$$

Proof. The result follows from taking the limit $\delta \to 1$ of (34) and again noting that $\pi_H(p_L^M) > D_H$ when (2) is violated. ■

The consequences of these results are worth emphasizing. They are that in the infinite horizon limit of the limit price regime, as the discount factor approaches one, the efficient type must force the inefficient type to make arbitrarily large losses in order to credibly signal that he is indeed efficient. This is because in this scenario, the benefits to H of perpetual incumbency approach infinity. This gives a very lopsided intertemporal profile of costs and benefits. The costs of signalling are all borne in a single period, whereas the benefits of effectively deterring entry accrue over an infinite number of periods.

For T and δ sufficiently large, it may well be that the set

$$A_H^+(T,t) \equiv \{p \in A_H(T,t) \cap \mathbb{R}_+\}$$

is empty. In other words, depending on the details of the product market, it may be that there are no positive prices that satisfy the H type's incentive compatibility constraint. As $T \to \infty$ and $\delta \to 1$, positive prices can only be secured if demand has a vertical asymptote at $p = 0$, that is, if $\lim_{p\to 0} Q(p) = \infty$. Even in this case, the equilibrium separating price may run afoul of the Areeda and Turner (1975) rule, requiring pricing above marginal cost.

Pooling. As is the case in preseparation periods in the separating equilibria, the constraints for the pooling equilibria do not depend explicitly on the remaining number of periods. It follows that the pooling equilibria are in fact not affected by the dynamic extension of the model (if they exist).

□ **Effects on the entry decisions.** In this subsection, I consider how the entrant's incentive to enter or stay out change when dynamics are introduced. I consider the necessary conditions for separation and pooling in turn.

Separating equilibrium. Recall that in the single-round setting, the necessary conditions for a separating limit price equilibrium are that $D_E(L) < F < D_E(H)$. The equivalent conditions in the T-period problem are that for $t = 1, \ldots, T-1$, it is the case that

$$D_E(L) < F\left(\frac{1-\delta}{1-\delta^{T-t+1}}\right) < D_E(H). \tag{36}$$

As the expression in parentheses is decreasing in the horizon T, the right-hand side inequality in (36) is trivially satisfied for any $T > 1$, if it is satisfied in the single-round setting. I therefore concentrate on the left-hand side inequality. There are two cases to consider. For $D_E(L) > (1-\delta)F$, there exists some period T_S^* such that for $t > T - T_S^*$, the necessary condition is violated and thus, there can be no separating limit price equilibrium. Note that an implication of this finding is that if one imposes conditions for a separating limit price equilibrium to be feasible for a long time horizon, a separating limit price equilibrium may not be feasible in the static setting.

For the case $D_E(L) < (1-\delta)F$, a long time horizon is not enough to rule out the possibility of separation with limit pricing. However for sufficiently patient players, entry cannot be deterred through separation because the necessary condition is violated. Define the following critical value of the discount factor:

$$\delta_S^* \equiv 1 - \frac{D_E(L)}{F}. \tag{37}$$

The following result then follows:

Proposition 7. For $\delta > \delta_S^*$, there can be no separation with limit pricing in the infinite horizon game.

In conclusion, when the future is sufficiently important (either because the horizon is very long or because the players are very patient), limit pricing may become infeasible altogether because the discounted postentry payoffs to the entrant are large enough to offset the entry fee F, even when competing against the efficient incumbent L. In this case, the only possible outcome is that of an immediate separation equilibrium, with each type of incumbent setting its monopoly price and the entrant entering against the inefficient incumbent (and staying out against the efficient incumbent).

Pooling equilibrium. For simplicity, define

$$R \equiv \mu D_E(H) + (1-\mu)D_E(L), \tag{38}$$

and recall that in the single-round setting, a necessary condition for the existence of a pooling limit price equilibrium is that $R < F$. In contrast, in the T-period setting, the equivalent necessary condition for a pooling equilibrium to be feasible in an arbitrary period $t = 1, \ldots, T-1$ is that

$$R\left(\frac{1-\delta^{T-t+1}}{1-\delta}\right) < F. \tag{39}$$

There are two cases to consider, depending on the magnitude of the left-hand side in the limit as the horizon becomes very distant. First, consider the case in which $R > (1-\delta)F$. In this case, there exists some period T_P^* such that for $t > T - T_P^*$, the necessary condition (29) for pooling is violated. In other words, if the remaining game is sufficiently long, then there can be no pooling equilibria. The reason for this result is simply that if the remaining number of periods is very large, then the prospect of earning R per period upon entry (even if discounted) is sufficient to offset the entry fee F. It is therefore not possible to discourage entry, even if pooling is feasible in the benchmark setting.

Next, consider the case in which $R < (1-\delta)F$. In this case, pooling may be feasible even for very long horizons, for some values of the discount factor δ. However, for sufficiently high patience, pooling can be ruled out even in this case. This is because the expected discounted postentry profits are so large that entry is attractive even for an entrant who cannot distinguish the two types of incumbent. Define the following critical value of the discount factor:

$$\delta_P^* \equiv 1 - \frac{R}{F}. \tag{40}$$

The following result can then be established:

Proposition 8. For $\delta > \delta_P^*$, there can be no pooling in equilibrium in the infinite horizon game.

For completeness, note that $\delta_P^* < \delta_S^*$. This implies that for some intermediate values of the discount factor, it may be possible to rule out pooling (and thus, delayed separation equilibria) and therefore conclude that the outcome will be that of an immediate separation equilibrium with limit pricing (if feasible).

4. Discussion

■ In this article, I analyzed a dynamic model of limit pricing and compared it with the outcome of a static single-round model. I showed that there are two regimes of interest. In one, the *monopoly price regime*, the only equilibrium satisfying the anchored D1 refinement involves separation in the first period on monopoly prices, that is, it is an immediate separation equilibrium. In the other, the *limit price regime*, pooling limit price equilibria and separating limit price equilibria (both satisfying the anchored D1 refinement) coexist, which leads to the possibility of delayed separation equilibria.[17] Although the dynamic pooling equilibrium is essentially a repetition of the static outcome, with both types of incumbent pooling on the efficient type's monopoly price, the latter may differ quantitatively from the separating limit price equilibrium in the static setting.

The dynamic nature of the game changes the incentives of the incumbent and the entrant in important ways. First, a long time horizon and patient players may significantly increase the cost of signalling, to the point that the firms must set negative prices (and incur arbitrarily large losses). Second, the prospect of large discounted sums of postentry profits (which are relevant when the horizon is long and the players are very patient) may make entry deterrence impossible to achieve. Last, in the infinite-horizon version of the game, the incumbent and the entrant may choose to collude upon entry. Such collusion in turn makes entry deterrence less attractive *ex ante* (for the incumbent) and less deterring (for the potential entrant).[18] In summary, the analysis shows that for a number of different reasons, when moving from a static setting to a dynamic setting, the practice of entry deterrence through limit pricing seems to become less viable. This suggests that the issue of potential limit pricing should perhaps not be a main concern for competition authorities. Also, it should be noted that only some of these results depend on the equilibrium selection approach adopted in this article and would remain valid across a number of different environments.[19]

The basic model I have considered consisted of a single incumbent firm and a single potential entrant. As is true in most models of limit pricing, the incumbent can be interpreted as a profit maximizing cartel rather than as a single firm. As regards the entrant, the assumption that there is only a single such firm makes the problem tractable. As surveyed by Carlton and Perloff (2004), the early literature on limit pricing did indeed consider a number (and often a continuum) of potential entrants. Two assumptions made this tractable. First, it was typically assumed that there was no coordination problem between entrants (i.e., they would enter in an orderly and continuous fashion if price was sufficiently high). Second, it was assumed that there was no signalling taking place. In the signalling-based limit pricing literature, the assumption of a single entrant is ubiquitous. I have chosen to keep with this modelling assumption in order to make my contribution directly comparable to this strand of the literature.[20]

Last, the model has been solved under the important assumption that upon entry, the firms compete under complete information, that is, there is no residual uncertainty about the incumbent's type. This is an assumption that would be interesting to relax, as it would open up the possibility of postentry predatory pricing, in the spirit of Benoit (1984). Nonetheless, a full analysis of such a setting seems very difficult to achieve, as one would have to consider repeated entry and exit both on and off the equilibrium path.

[17] Note that in Saloner (1984), delayed entry is the outcome of a strategic choice by the entrant, whereas in the present article, it is the outcome of the parties coordinating on a particular equilibrium among many.

[18] For further discussion of the effects of postentry collusion, see the web Appendix.

[19] In related work (available upon request), I show that in an infinite-horizon version of the model in which the incumbent's type evolves stochastically over time, when the players become sufficiently patient, entry cannot be deterred. Similar results are also likely to hold in the setting considered by Saloner (1984).

[20] Kalish, Hartzog, and Cassidy (1978) and references therein study some of the complications of the presence of multiple (mutually aware) entrants.

Appendix

This appendix contains the main proofs omitted in the main text.

Proof of Lemma 2 (efficient incumbent's incentive constraints). I first derive the condition for the separating equilibrium price. The incentive compatibility constraints for the L type are given by

$$\pi_L(p^*_{1,L}) + \sum_{i=2}^{T} \delta^{i-1}\pi_L(p^M_L) \geq \pi_L(p^M_L) + \sum_{i=2}^{T} \delta^{i-1} D_L \tag{A1}$$

$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_L(p^*_i) + \delta^{K+1}\pi_L(p^*_{K+2,L}) + \sum_{i=K+3}^{T} \delta^{i-1}\pi_L(p^M_L)$$
$$\geq \sum_{i=1}^{M+1} \delta^{i-1}\pi_L(p^*_i) + \delta^{M+1}\pi_L(p^M_L) + \sum_{i=M+3}^{T} \delta^{i-1} D_L \tag{A2}$$

for $0 \leq M \leq K = 0, 1, ..., T-3$. The first constraint (A1) reduces to

$$\pi_L(p^*_{1,L}) \geq \left(1 - \frac{\delta - \delta^T}{1-\delta}\right)\pi_L(p^M_L) + \left(\frac{\delta - \delta^T}{1-\delta}\right) D_L. \tag{A3}$$

Next, evaluate (A2) at $M = K$ and rearrange to get

$$\pi_L(p^*_{K+2,L}) \geq \left(1 - \frac{\delta - \delta^{T-K-1}}{1-\delta}\right)\pi_L(p^M_L) + \left(\frac{\delta - \delta^{T-K-1}}{1-\delta}\right) D_L, \tag{A4}$$

which determines the separating prices. Next, evaluate (A2) at two arbitrary consecutive periods $M = K - j$ and $M = K - j - 1$, respectively, with $j = 1, ..., K-1$. These yield

$$\delta^{K-j+1}\pi_L(p^*_{K-j+2}) - \left(\delta^{K-j+1} - \sum_{i=K+3}^{T} \delta^{i-1}\right)\pi_L(p^M_L) \geq \tag{A5}$$

$$\sum_{i=K-j+3}^{T} \delta^{i-1} D_L - \delta^{K+1}\pi_L(p^*_{K+2,L}) - \sum_{i=K-j+3}^{K+1} \delta^{i-1}\pi_L(p^*_i)$$

$$\delta^{K-j}\pi_L(p^*_{K-j+1}) - \left(\delta^{K-j} - \sum_{i=K+3}^{T} \delta^{i-1}\right)\pi_L(p^M_L) \geq \tag{A6}$$

$$\sum_{i=K-j+2}^{T} \delta^{i-1} D_L - \delta^{K+1}\pi_L(p^*_{K+2,L}) - \sum_{i=K-j+3}^{K+1} \delta^{i-1}\pi_L(p^*_i) - \delta^{K-j+1}\pi_L(p^*_{K-ji+2}).$$

Substituting (A5) in (A6), rearranging and reducing yields

$$\pi_L(p^*_{K-j+1}) \geq (1-\delta)\pi_L(p^M_L) + \delta D_L. \tag{A7}$$

Last, if the equilibrium requires pooling in only the first period, then it must be that

$$\pi_L(p^*_1) \geq \left(1 - \sum_{i=3}^{T} \delta^{i-1}\right)\pi_L(p^M_L) + \sum_{i=2}^{T} \delta^{i-1} D_L - \delta\pi_L(p^*_{K+2,L}). \tag{A8}$$

Substituting for the value of $\pi_L(p^*_{K+2,L})$ given by (A4) and rearranging yields

$$\pi_L(p^*_1) \geq (1-\delta)\pi_L(p^M_L) + \delta D_L. \tag{A9}$$

This completes the proof. ■

Proof of Lemma 3 (inefficient incumbent's incentive constraints). The constraints are as follows:

$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p^*_i) + \delta^{K+1}\pi_H(p^M_H) + \sum_{i=K+3}^{T} \delta^{i-1} D_H \geq \sum_{i=1}^{M+1} \delta^{i-1}\pi_H(p^*_i) + \delta^{M+1}\pi_H(p^M_H) + \sum_{i=M+3}^{T} \delta^{i-1} D_H,$$
$$0 \leq M < K = 0, ..., T-3 \tag{A10}$$

$$\pi_H(p^M_H) + \sum_{i=2}^{T} \delta^{i-1} D_H \geq \pi_H(p^*_{1,L}) + \delta\pi_H(p^M_H) + \sum_{i=3}^{T} \delta^{i-1} D_H \tag{A11}$$

$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1} D_H \geq \sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_{K+2,L}^*) + \delta^{K+2}\pi_H(p_H^M)$$
$$+ \sum_{i=K+4}^{T} \delta^{i-1} D_H, \quad K = 0, ..., T-4 \tag{A12}$$

$$\pi_H(p_H^M) + \sum_{i=K+2}^{T} \delta^{i-1} D_H \geq \pi_H(p_{1,L}^*) + \sum_{i=2}^{K+2} \delta^{i-1}\pi_H(p_L^M) + \delta^{K+2}\pi_H(p_H^M) + \sum_{i=K+4}^{T} \delta^{i-1} D_H, \quad K = 0, ..., T-4 \tag{A13}$$

$$\sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1} D_H \geq \sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_{K+2,L}^*) + \sum_{i=K+3}^{M+2} \delta^{i-1}\pi_H(p_L^M)$$
$$+ \delta^{M+2}\pi_H(p_H^M) + \sum_{i=M+4}^{T} \delta^{i-1} D_H, \quad 0 \leq K < M = 0, ..., T-4 \tag{A14}$$

$$\pi_H(p_H^M) + \sum_{i=2}^{T} \delta^{i-1} D_H \geq \pi_H(p_{1,L}^*) + \sum_{i=2}^{T-1} \delta^{i-1}\pi_H(p_L^M) + \delta^{T-1}\pi_H(p_H^M) \tag{A15}$$

$$\sum_{i=1}^{K+1} \delta^{j-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_H^M) + \sum_{i=K+3}^{T} \delta^{i-1} D_H \geq \sum_{i=1}^{K+1} \delta^{i-1}\pi_H(p_i^*) + \delta^{K+1}\pi_H(p_{K+2,L}^*) + \sum_{i=K+3}^{T-1} \delta^{i-1}\pi_H(p_L^M)$$
$$+ \delta^{T-1}\pi_H(p_H^M), \quad K = 0, ..., T-4. \tag{A16}$$

These sets of constraints will be explained in turn. Roughly, the H type's off-equilibrium behavior can be described by the sequence *pool-mimic-reveal*. That is, first H pools whenever the L type pools, then the H type mimics L's behavior for some number of periods, and then he reveals his type, subsequently earning duopoly profits following entry by E. The first set (A10) considers the possibility of the H type revealing his type by setting the monopoly price earlier than the period in which the L type separates. These constraints will determine the pooling constraints for the H type. Next, the constraints (A11) and (A12) consider the H type mimicking the L type for a single period, in the cases of no prior pooling and an arbitrary number of prior periods with pooling, respectively. Constraints (A13) and (A14) consider the H type mimicking the L type for a number of periods, in the cases of no prior pooling and an arbitrary number of prior periods with pooling, respectively. Last, constraints (A15) and (A16) consider the possibility of the H type perpetually mimicking the L type, again in the cases of no prior pooling and an arbitrary number of prior periods with pooling, respectively.

The first equations in parts (i) and (ii) of the lemma follow from the constraints (A10) and the same steps as those leading to the incentive compatibility constraints for the L type.

The next step is to order the magnitudes of the right-hand sides of constraints (A11)–(A16). Straightforward comparison shows that the order depends on whether or not

$$(1-\delta)\pi_H(p_H^M) + \delta D_H \geq \pi_H(p_L^M). \tag{A17}$$

If (A17) is satisfied, then (A11)–(A12) imply (A13)–(A16), whereas if (A17) is violated, then (A11)–(A14) are implied by (A15)–(A16). Note that condition (A17) is in fact just a restatement of condition (2), that is, the condition that delineates the monopoly price regime and the limit price regime, respectively.

The incentive compatibility constraints, if (A17) is satisfied, are thus (A12), which reduce to

$$\pi_H(p_{K+2,L}^*) \leq (1-\delta)\pi_H(p_H^M) + \delta D_H \tag{A18}$$

for $K = 0, ..., T-4$, whereas the equivalent constraint for the first period follows from (A11). If (A17) is violated, then the relevant incentive compatibility constraints are (A16), which reduce to

$$\pi_H(p_{K+2,L}^*) \leq \left(1-\delta^{T-K-2}\right)\pi_H(p_H^M) + \left(\frac{\delta - \delta^{T-K-1}}{1-\delta}\right) D_H - \left(\frac{\delta - \delta^{T-K-2}}{1-\delta}\right)\pi_H(p_L^M) \tag{A19}$$

for $K = 0, ..., T-4$, whereas the equivalent constraint for the first period follows from (A15). ■

Proof of Proposition 4 (existence of separating limit price equilibrium). To prove the proposition, some preliminary notation and results are needed. As $D_H = \pi_H(p)$ for some p, then by Assumptions 6 and 7, the set A_H is nonempty and contains at most two points given by $\widehat{\alpha}$ and $\widehat{\beta}$, where $\widehat{\alpha} < \infty$ and $\widehat{\beta} \leq \infty$. For later use, note that by definition,

$$\pi_H(\widehat{\alpha}) = \delta D_H + (1-\delta)\pi_H(p_H^M) = \pi_H(\widehat{\beta}). \tag{A20}$$

Observe that $p_L^* < p_L^M < p_H^M < \widehat{\beta}$. In conclusion, for the H type's incentive compatibility constraint to hold, it must be that

$$p_L^* \leq \widehat{\alpha}. \tag{A21}$$

In other words, in order for the high-cost incumbent to be willing to tell the truth, the low-cost incumbent's strategy must be sufficiently low.

Next, as $D_L = \pi_L(p)$ for some p, then by Assumptions 6 and 7, the set A_L is nonempty and contains at most two points, given by α_0 and β_0, where $\alpha_0 < \infty$ and $\beta_0 \leq \infty$. By definition, it is the case that

$$\pi_L(\alpha_0) = \delta D_L + (1-\delta)\pi_L(p_L^M) = \pi_L(\beta_0). \tag{A22}$$

This means that for the low-cost incumbent to be willing to engage in costly signalling, the separating equilibrium price must be high enough.

Last, the following result is needed:

Lemma 15 (relative efficiency of types).

(i) $\pi_L(p) - \pi_H(p)$ is strictly decreasing in p, and (ii) $p_H^M > p_L^M$.

Proof. See the web Appendix. ■

Next, I turn to the proof of the proposition:

(i) Solving (16) and (A20) for D_L and D_H, respectively, substituting into (27) and rearranging, yields

$$\pi_L(p_L^M) - \pi_H(p_H^M) > \pi_L(\alpha_0(T,t)) - \pi_H(\widehat{\alpha}). \tag{A23}$$

Adding and subtracting $\pi_H(p_L^M)$ yields

$$\pi_L(p_L^M) + \pi_H(p_L^M) + \left[\pi_H(p_L^M) - \pi_H(p_H^M)\right] > \pi_L(\alpha_0(T,t)) - \pi_H(\widehat{\alpha}). \tag{A24}$$

By the definition of p_H^M, it follows that $\pi_H(p_L^M) - \pi_H(p_H^M) \leq 0$. It thus follows from (A24) that

$$\pi_L(p_L^M) + \pi_H(p_L^M) > \pi_L(\alpha_0(T,t)) - \pi_H(\widehat{\alpha}). \tag{A25}$$

As $\alpha_0(T,t) \leq p_L^M$, it follows by Lemma 15 that

$$\pi_L(\alpha_0(T,t)) - \pi_H(\alpha_0(T,t)) \geq \pi_L(p_L^M) - \pi_H(p_L^M). \tag{A26}$$

Combined with (A25), this implies that $\pi_H(\alpha_0(T,t)) < \pi_H(\widehat{\alpha})$. Finally, $\alpha_0(T,t) \leq p_H^M$ and $\widehat{\alpha} \leq p_H^M$ and, therefore, it follows by Assumption 6 that $\alpha_0(T,t) < \widehat{\alpha}$.

(ii) Solving (16) and (26) for D_L and D_H, respectively, substituting into (28) and rearranging, yields

$$\pi_L(p_L^M) - \pi_H(p_H^M) > \pi_L(\alpha_0(T,t)) - \pi_H(\widehat{\alpha}(T,t)). \tag{A27}$$

Similar steps as in (i) then complete the proof. ■

Proof of Proposition 5 (uniqueness of separating limit price equilibrium). First, condition (2), which delineates the two regimes, holds if and only if $\widehat{\alpha}(T,t) \geq p_L^M$. To see this, note that from (26), it follows that

$$\delta D_H = \left(\frac{1-\delta}{1-\delta^{T-t}}\right)\left[\pi_H(\widehat{\alpha}(T,t)) - (1-\delta^{T-t})\pi_H(p_H^M) + \left(\frac{\delta-\delta^{T-t}}{1-\delta}\right)\pi_H(p_L^M)\right]. \tag{A28}$$

Substituting this in (2) yields

$$\pi_H(\widehat{\alpha}(T,t)) \leq \pi_H(p_L^M). \tag{A29}$$

As $\widehat{\alpha}(T,t) < p_H^M$ and $p_L^M < p_H^M$, the result follows from the inequality and Assumption 6. Next, I make use of this result to prove the proposition. (i) Suppose that $\alpha_0(T,t) < \widehat{\alpha}(T,t) \leq p_L^M$ and let p' satisfy $p_L^* < p' < \widehat{\alpha}(T,t)$. Whichever strategy E picks, it is a strictly dominated strategy for H to choose p'. If $s_E(p') = 1$, then because $p' < \widehat{\alpha}(T,t) \leq p_L^M \leq p_H^M$, Assumption 6 implies that the H type can benefit from switching to p_H^M, thereby earning $\pi_H(p_H^M) - \pi_H(p') > 0$. Next, suppose that $s_E(p') = 0$. In equilibrium, the H type should set the price p_H^M and can never earn more out of equilibrium than by playing his optimal off-equilibrium strategy. However, the first element of this strategy is precisely given by $\widehat{\alpha}(T,t)$.

It follows that the H type is better off by switching from p' to $\widehat{\alpha}(T,t)$. After deleting the price p' from the H type's strategy set, E must set $s_E(p')=0$, as p' could only have been set by the L type. However, because $p' < \widehat{\alpha}(T,t) \le p_L^M$, it follows from Assumption 6 that the L type is better off by increasing his price to $\widehat{\alpha}(T,t)$. The proof of (ii) follows similar steps as that of (i). ■

Proof of Lemma 8 (incumbents' incentive constraints). The incentive compatibility constraints for the L type are given by[21]:

$$\sum_{i=1}^{T-1}\delta^{i-1}\pi_L(p_i^*)+\delta^{T-1}\pi_L(p_L^M) \ge \pi_L(p_L^M)+\sum_{i=2}^{T}\delta^{i-1}D_L \tag{A30}$$

$$\sum_{i=1}^{T-1}\delta^{i-1}\pi_L(p_i^*)+\delta^{T-1}\pi_L(p_L^M) \ge \sum_{i=1}^{K+1}\delta^{i-1}\pi_L(p_i^*)+\delta^{K+1}\pi_L(p_L^M)+\sum_{i=K+3}^{T}\delta^{i-1}D_L \tag{A31}$$

for $K=0,1,\ldots,T-3$. The set of constraints (A31), (one for each K) compares the equilibrium strategy with a strategy that pools until (and including) period $K+1$ and deviates in period $K+2$. Solving (A30) for $\pi_L(p_1^*)$, yields

$$\pi_L(p_1^*) \ge (1-\delta^{T-1})\pi_L(p_L^M)+\sum_{i=2}^{T}\delta^{i-1}D_L-\sum_{i=3}^{T-1}\delta^{i-1}\pi_L(p_i^*)-\delta\pi_L(p_2^*). \tag{A32}$$

Evaluating (A31) at $K=0$ and rearranging yields

$$\delta\pi_L(p_2^*) \ge \sum_{i=K+3}^{T}\delta^{i-1}D_L+\delta\pi_L(p_L^M)-\sum_{i=3}^{T-1}\delta^{i-1}\pi_L(p_i^*). \tag{A33}$$

Substituting this in (A32) and rearranging gives

$$\pi_L(p_1^*) \ge (1-\delta)\pi_L(p_L^M)+\delta D_L.$$

For arbitrary K, (A31) reduces to

$$\sum_{i=K+2}^{T-1}\delta^{i-1}\pi_L(p_i^*)+\delta^{T-1}\pi_L(p_L^M) \ge \delta^{K+1}\pi_L(p_L^M)+\sum_{i=K+3}^{T}\delta^{i-t}D_L. \tag{A34}$$

Straightforward manipulation yields that this inequality can be rewritten as

$$\sum_{i=0}^{T-K-3}\delta^{i}\pi_L(p_{i+K+2}^*)+\delta^{T-K-2}\pi_L(p_L^M) \ge \pi_L(p_L^M)+\sum_{i=0}^{T-K-3}\delta^{i+1}D_L. \tag{A35}$$

In particular, this implies that

$$\pi_L(p_{K+2}^*) \ge \left(1-\delta^{T-K-2}\right)\pi_L(p_L^M)+\sum_{i=0}^{T-K-3}\delta^{i+1}D_L-\sum_{i=2}^{T-K-3}\delta^{i}\pi_L(p_{i+K+2}^*)-\delta\pi_L(p_{K+3}^*). \tag{A36}$$

However, the constraint on $\pi_L(p_{K+3}^*)$ is in turn given by

$$\delta\pi_L(p_{K+3}^*) \ge \left(\delta-\delta^{T-K-2}\right)\pi_L(p_L^M)+\sum_{i=1}^{T-K-3}\delta^{i+1}D_L-\sum_{i=2}^{T-K-3}\delta^{i}\pi_L(p_{i+K+2}^*). \tag{A37}$$

Substituting this back in (A36) and rearranging yields the following constraints:

$$\pi_L(p_{K+2}^*) \ge (1-\delta)\pi_L(p_L^M)+\delta D_L \tag{A38}$$

for $K=0,1,\ldots,T-3$. Similar steps yield the equivalent constraints for the H type. ■

Proof of Proposition 11 (cost of signalling with infinite horizon). For $\lim_{T\to\infty}\pi_H(\widehat{\alpha}(T,t)) \le 0$ to hold, it follows from (34) that the inequality

$$\pi_H(p_L^M)-D_H \ge \left(\frac{1-\delta}{\delta}\right)\pi_H(p_H^M) \tag{A39}$$

[21] It is, without loss of generality, to consider a deviation in period 1, because if there is pooling in periods $s=1,\ldots,t-1$, then the period t problem is essentially the same as that faced in period 1.

must hold. This inequality can be rewritten as

$$\delta \geq \frac{\pi_H(p_H^M)}{\pi_H(p_H^M) - D_H + \pi_H(p_L^M)} \equiv \delta^{**}. \tag{A40}$$

Next, note that if $\pi_H(p_L^M) > D_H$, then (2) can be rewritten as

$$\delta \leq \frac{\pi_H(p_H^M) - \pi_H(p_L^M)}{\pi_H(p_H^M) - D_H} \equiv \delta^{*}. \tag{A41}$$

From Assumption 7, it follows that the violation of (2) is a sufficient (but not necessary) condition for $\pi_H(p_L^M) > D_H$ to hold. Last, note that $\delta^* \geq \delta^{**}$ if and only if $\pi_H(p_L^M) > D_H$, which is implied by the assumption that (2) is violated. ■

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